A RATE-INDEPENDENT THERMOMECHANICAL CONSTITUTIVE MODEL FOR FIBROUS REINFORCEMENTS

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ABSTRACT: In Liquid Composite Moulding (LCM) processes, the constitutive behaviour of fibrous reinforcements has a strong bearing on the choice of manufacturing parameters and final part properties. In many LCM processes, the fibrous preform is subjected to loading and unloading, the latter also occurring during filling and post-filling phases of the manufacturing process. Fibrous materials display inelastic behaviour with rate-dependent and rate-independent components and this must be modelled accurately over several load-unload cycles in order to accurately simulate such processes. An important feature of the material behaviour is its unchanging response to successive load cycles once a large number of load cycles have been applied. Inelastic effects such as fibre-fibre frictional sliding occur during loading as well as unloading and the inelastic deformation remaining after successive cycles appears unchanged. The model presented is developed within a thermomechanical framework and reproduces such behaviour using a single internal variable to account for inelasticity. It is compared to cyclic loading experiments and serves as a starting point for the incorporation of effects such as cyclic softening and rate-effects through additional internal variables.

KEYWORDS: Compaction model, fibrous materials, permanent deformation, hysteresis

INTRODUCTION

The term Liquid Composite Moulding (LCM) is used to describe a number of composite material manufacturing processes in which a dry reinforcement fibrous preform is placed in a mould and then impregnated with liquid resin. The mould construction and the manner in which the resin is forced through the fibrous preform distinguish the various LCM processes from one another.

Compaction of a fibrous preform is common among all the LCM processes as a means of achieving a high volume fraction of the final composite part and associated benefits. In rigid mould processes such as RTM (Resin Transfer Moulding) and CRTM (Compression Resin Transfer Moulding), deformation of the mould is not a factor and knowledge of the constitutive behaviour of the fibrous reinforcement is critical in the prediction of tooling forces, process parameter selection and tooling designs. Across all the LCM processes, some benefit may be obtained from the application of cyclic loads to the dry reinforcement before the final compaction (a process known as "debulking") so that a desired volume fraction is achieved at lower compaction forces. Rate-dependent and rate-independent inelastic effects need to be taken into account in order for the benefits of cyclic compaction to be quantified.

Unlike with the rigid-mould processes, the constitutive response of the preform contributes significantly to mould deformation in RTM-Light and VARI (Vacuum Assisted Resin Infusion). This in turn determines the tolerances achievable using flexible-mould processes and their viability as cost-effective alternatives to rigid mould processes. Significant mould and preform deformations still occur after the initial compaction as a result of resin flow. Resin flow initially relieves the preform of compaction stress, but the preform will typically become reloaded during the post-filling stage as resin pressures equilibrate. For this reason one requires constitutive models which account for several cycles of loading and not simply a single initial compaction.

The constitutive model development presented in this paper is a step towards producing a full-scale simulation of a generic LCM process. For this purpose, the model must capture the salient features present in compaction experiments, including time and load-history dependence, yet be practical to characterize experimentally. The accuracy of the model within full-scale simulations should not be undermined by excessive complexity or computational expense. Finally, the model must have a physical basis, allowing it to provide insight into the physical processes which contribute to the constitutive behaviour.

To date, several approaches have been adopted to describe the constitutive behaviour of fibrous reinforcements. Van Wyk [1] pioneered a probability-distribution-based micromechanical model from which a non-linear elastic bulk response was obtained. One extension of note to this approach was that of Carnaby and Pan [2], where frictional effects were incorporated to produce a hysteretic response. Such probability-based models do not use explicit geometric representations, so are computationally inexpensive. Other micromechanical models such as [3] involve explicit geometric representations of fabrics at the meso-scale. Individual fibres within the tow could be modelled similarly to account for interactions within the tow, however at present this appears computationally prohibitively expensive.

An alternative to the micromechanical approaches mentioned above involves the treatment of fibrous materials as continua. Inelastic effects can be considered within a continuum mechanics framework using such hypothetical components as dashpots, springs and friction blocks, without the computational expense associated with micromechanical models. Several models have reproduced aspects of the permanent deformation and time dependent response observed in experiments [4,5]. The thermomechanical model proposed in this paper utilizes a continuum approach, but lies within a framework which ensures that thermodynamic requirements are satisfied. Physical processes which contribute to the material behaviour are described using state variables, their evolution governed by the laws of thermodynamics. Within the continuum-based approach, two models of note are those of Kelly [6] and Comas-Cardona et. al [5]. The basic non-linear hysteretic response is accounted for, but the behaviour upon unloading is entirely elastic.

THERMOMECHANICAL MODELLING OF FIBROUS MATERIALS

Modelling Framework

For isothermal deformations, the power of stresses applied may be expressed as shown in [7],

$$\boldsymbol{\sigma}:\boldsymbol{d} = \dot{\Psi} + \Phi \, , \Phi \geq 0 \tag{1}$$

$\boldsymbol{\sigma}$ and $\boldsymbol{d}$ are the stress and deformation rate tensors respectively, their inner product being the rate of applied work. The constitutive behaviour of the material is defined by two potential functions, the Helmholtz free energy potential Ψ and the dissipation rate, Φ. Ψ gives the amount of energy stored within the material, while Φ is the rate at which energy is lost from the material through dissipative processes such as friction and heat transfer. The second law of thermodynamics states that Φ must be non-negative.

The definition of potential functions Ψ and Φ are a point of difference between the thermomechanical approach and classical models in plasticity or viscoplasticity, which define material behaviour through a combination of yield functions and flow rules. In the thermomechanical framework, these functions follow directly from the definitions of Ψ and Φ and thermodynamic permissibility is imposed.

Fibrous materials

A number of physical processes contribute to the complex behaviour of fibrous materials. Several authors [1, 2] acknowledge the contribution of fibre bending to the material behaviour, whereby strain energy is accumulated in the material. This energy may also become *frozen* into the material if fibres become locked into bent configurations and, due to friction, do not fully un-bend upon unloading; this manifests itself as a permanent deformation upon unloading. Fibres exhibit frictional behaviour, providing an avenue for dissipation. Rate effects may also arise from dissipative time-dependent rearrangement of fibres within the tow. Unlike traditional inelastic materials, fibrous materials do not exhibit a clear yield point; inelastic processes such as fibre-fibre sliding and fibre-rearrangement appear to occur instantaneously and continuously with load.

The repeatable state

When subjected to cyclic compaction loading, for example repeatedly loading to some given volume fraction and unloading again, fibrous materials exhibit a debulking response. That is, the material softens (maximum stress reduces) with each successive load cycle, with the extent of additional softening decreasing with each cycle. Eventually, the response reaches a single hysteresis loop which appears unchanging with successive cycles. Key features of this load-cycle-invariant behaviour include significant hysteresis as well as an apparently constant value of permanent deformation remaining after successive cycles. The goal of this study is to develop a model of this repeatable behaviour as a precursor to a more comprehensive model, where cyclic softening and rate-effects are superimposed through the use of additional internal variables.

1-INTERNAL VARIABLE MODEL

For simplicity, a single internal variable is considered. Total strain ε_{ij} is additively decomposed into elastic strain, ε_{ij}^{e}, and the inelastic strain, ε_{ij}^{i}. Inelastic effects are accounted for through evolution of the inelastic strain, which is equivalent to the internal variable α_{ij}.

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{i} \qquad \varepsilon_{ij}^{e} = \varepsilon_{ij} - \alpha_{ij} \qquad \varepsilon_{ij}^{i} = \alpha_{ij} \tag{2}$$

One assumes that the material is *decoupled*, so that the elastic behaviour of the material is independent of the value of the inelastic strain (i.e of plastic effects). As such, the free energy potential function takes the following form:

$$\Psi(\varepsilon_{ij},\alpha_{ij}) = \Psi_1(\varepsilon_{ij}-\alpha_{ij}) + \Psi_2(\alpha_{ij}) \;\; \textit{or equivalently,} \;\; \Psi(\varepsilon_{ij},\alpha_{ij}) = \Psi_1(\varepsilon_{ij}^{e}) + \Psi_2(\varepsilon_{ij}^{i}) \tag{3}$$

Ψ_1 represents the strain energy associated with deformations that are reversible with unloading and Ψ_2 represents locked energy associated with inelastic deformation.

The dissipation rate is homogeneous of degree 1 in the inelastic strain rate $\dot{\alpha}_{ij}$, giving rise to rate-independent inelastic behaviour.

$$\Phi = \frac{\partial \Phi}{\partial \dot{\alpha}_{ij}} \dot{\alpha}_{ij} \tag{4}$$

Substituting the free energy (3) and dissipation (4) functions into (1), one has:

$$0 = \left(\frac{\partial \Psi}{\partial \varepsilon_{ij}} - \sigma_{ij}\right)\dot{\varepsilon}_{ij} + \left(\frac{\partial \Psi}{\partial \alpha_{ij}} + \frac{\partial \Phi_1}{\partial \dot{\alpha}_{ij}}\right)\dot{\alpha}_{ij} \,, \tag{5}$$

The coefficient term of $\dot{\varepsilon}_{ij}$ is independent of the rates $\dot{\varepsilon}_{ij}$ and $\dot{\alpha}_{ij}$. To proceed, one may appeal to Ziegler's Hypothesis [8], from which it follows that the coefficient term of $\dot{\alpha}_{ij}$, while clearly independent of $\dot{\varepsilon}_{ij}$, can be treated as independent also of $\dot{\alpha}_{ij}$. Thus, one has the following relations:

$$\begin{aligned} &\sigma_{ij} = \frac{\partial \Psi}{\partial \varepsilon_{ij}} = \frac{\partial \Psi_1(\varepsilon_{ij}-\alpha_{ij})}{\partial(\varepsilon_{ij}-\alpha_{ij})}, \qquad &&\left(-\bar{\chi}_{ij} + \chi_{ij}\right) = 0 \;\; \textit{for inelasticity} \\ &\bar{\chi}_{ij} = -\frac{\partial \Psi}{\partial \alpha_{ij}} = \frac{\partial \Psi_1(\varepsilon_{ij}-\alpha_{ij})}{\partial(\varepsilon_{ij}-\alpha_{ij})} - \frac{\partial \Psi_2(\alpha_{ij})}{\partial \alpha_{ij}} &&\chi_{ij} = \frac{\partial \Phi}{\partial \dot{\alpha}_{ij}} = \Phi \dot{\alpha}_{ij}^{-1} \end{aligned} \tag{6}$$

$\bar{\chi}_{ij}$ is the termed the generalized *quasi-conservative stress* and χ_{ij} is the generalized *dissipative stress*. Ψ_2' is equivalent to the shift stress ρ_{ij} in classical plasticity theory and represents the shift in the yield surface in generalized stress space causing a kinematic hardening effect.

Expressing the first relation in (6) in rate form, one has:

$$\dot{\sigma}_{ij} = \frac{\partial}{\partial t}\left(\frac{\partial \Psi_1(\varepsilon_{ij}-\alpha_{ij})}{\partial(\varepsilon_{ij}-\alpha_{ij})}\right) = K^e_{ijkl}\dot{\varepsilon}_{kl} + K^\alpha_{ijkl}\dot{\alpha}_{kl}$$
$$K^e_{ijkl} = \frac{\partial^2 \Psi_1(\varepsilon_{ij}-\alpha_{ij})}{\partial(\varepsilon_{ij}-\alpha_{ij})\partial(\varepsilon_{kl}-\alpha_{kl})} \qquad K^\alpha_{ijkl} = -K^e_{ijkl}\ , \tag{7}$$

from which it is evident that the elastic stiffness matrix K^e_{ijkl} derives directly from the first term of the free energy function prescribed. In order to incorporate non-linear elastic behaviour and non-linear accumulation of frozen energy for increasing inelastic strain, power laws are prescribed for the free energy functions:

$$\Psi_1(\varepsilon_{ij}-\alpha_{ij}) = \frac{1}{m}E(\hat{\varepsilon}^e)^m \qquad \Psi_2(\alpha_{ij}) = \frac{1}{p}H(\hat{\alpha})^p$$
$$\textit{Effective elastic strain: } \hat{\varepsilon}^e(\varepsilon^e_{ij}) = \sqrt{\frac{2}{3}\varepsilon^e_{ij}\varepsilon^e_{ij}} \quad \textit{Accumulated inelastic strain: } \hat{\alpha}(\alpha_{ij}) = \sqrt{\frac{2}{3}\alpha_{ij}\alpha_{ij}} \tag{8}$$

A dissipation rate of the form below is chosen:

$$\Phi = \mathrm{sgn}(\dot{\hat{\alpha}}) f(\sigma_{ij})\,\dot{\alpha}_{ij}$$
$$f(\sigma_{ij}) = k\hat{\sigma}^{v-1}(\sigma^* - \hat{\sigma})^w \sigma_{ij}\ , \qquad 0 \le \hat{\sigma} \le \sigma^*\ , \quad \hat{\sigma}\textit{: Von-Mises stress} \tag{9}$$

$f_{ij}(\sigma_{ij})$ is positive definite for effective stresses within the limits $[0, \sigma^*]$, so that the dissipation rate is non-negative and the second law of thermodynamics is necessarily satisfied within the stress limits specified. v and w are positive powers, such that $f_{ij}(\sigma_{ij})$ is a function with roots at the effective stress limits 0 and σ^*. The dependence on stress is consistent with the frictional behaviour of fibrous materials and the stress limit σ^*, a material parameter, represents the stress at which the dissipation rate is zero, a requirement for perpetual inelastic behaviour as loading ends and unloading commences. From (6) and (9), the yield stresses for positive and negative increments in the inelastic strain α are given by:

$$\sigma_{ij} = \rho_{ij}(\alpha_{ij}) + \mathrm{sgn}(\dot{\hat{\alpha}}) f_{ij}(\sigma_{ij})$$
$$\textit{for } \hat{\sigma} = \sigma^*: \quad f_{ij}(\sigma_{ij}) = 0 \cdot \mathbb{I} \quad \rightarrow \quad \sigma_{+ij} = \sigma_{-ij} = \rho_{ij}(\alpha_{ij}) \tag{10}$$

Where the subscript of σ denotes the load direction, or sign of the increment in accumulated inelastic strain $d\hat{\alpha}$. At an effective stress of σ^*, relation (10a) is necessarily satisfied for inelastic loading and unloading, so that there is no zone of elastic behaviour as unloading commences.

RESULTS AND DISCUSSION

The proposed model for fibrous materials exhibiting a repeatable response to cyclic loading is compared with data from past experiments carried out by the authors and others at the Centre for Advanced Composite Materials, University of Auckland and the Polymers and Composites Lab, L'Ecole des Mines de Douai. The experiments were conducted as described in the table below:

Table 1: Cyclic compaction experiment details

	Target final / initial v_f	Compaction speed (mm/min)	cycles
CFRM Glass	0.60 / 0.30	5	20
CSM Glass	0.42 / 0.24	25	60
Twill Weave Glass	0.64 / 0.32	2	40

The model is fitted to the stress versus volume fraction curve for the last load-unload cycle for each experiment. Material parameters were ascertained in a simple way; parameters for the first free energy function and generalized dissipative stress were prescribed, then optimal values for the locked energy function computed by minimizing the objective function:

$$R = W_{load}\left(\|\overline{v_f}\|_n + \|\bar{\tilde{\sigma}}\|_n\right)_{load} + W_{unload}\left(\|\overline{v_f}\|_n + \|\bar{\tilde{\sigma}}\|_n\right)_{unload}, \quad given: \ \hat{\rho} = \sigma^* \tag{11}$$

W: Weighting factor; $\quad \| \quad \|_n$ *denotes n^th^ norm; overbar denotes residual*

The parameters obtained in this way are listed in Table 2. Results are shown in Figure 1. The model shows excellent agreement with experiments for all three material architectures tested.

Table 2: Model Parameters – Italicized values obtained through optimization

	Free Energy Ψ_1		Free Energy Ψ_2		Dissipation Φ			
	E	m	H	p	σ^*	k (load/unload)	v	w
CFRM	2e12 Pa	5	***3.6e-4 E***	***18***	170 kPa	0.25 / 0.25	1	1
CSM	5e8 Pa	4	***1e-4 E***	***13***	45 kPa	0.23 / 0.23	1	1
Twill Weave	2e10 Pa	3	***0.65 E***	***20***	77 kPa	0.14 / 0.14	1	2

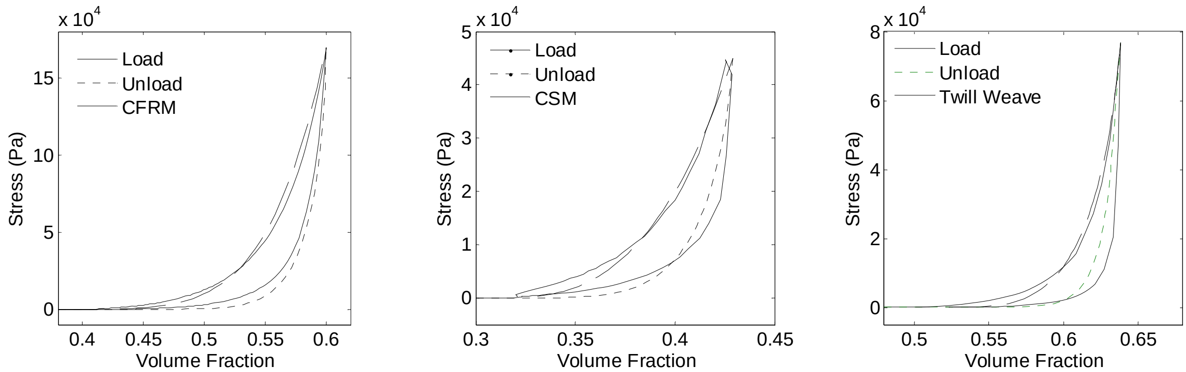

Figure 1: Stress against Volume Fraction - experimental and model data for CFRM, CSM and Twill Weave samples

Note that the model exhibits inelastic behaviour for both loading and unloading and the response to successive load cycles repeats. A logical extension to the model would be the inclusion of an additional internal variable to account for the cyclic softening behaviour which precedes repeatable behaviour during cyclic compaction experiments. Further studies may be carried out using the current model to investigate the evolution in inelastic strain and locked energy as the material is loaded and unloaded. Further work needs to be carried out in seeking improved correlation between simulation and experiment by extending the parameter estimation approach employed for Ψ_2 to Ψ_1 and Φ.

CONCLUSIONS

A rate-independent thermomechanical model for fibrous materials exhibiting repeatable hysteretic behaviour has been presented. A single internal variable was used to account for energy storage through fibres becoming locked in strained configurations. Continuous frictional inelastic behaviour implies that dissipative deformation is not permitted at the effective stress limit defined by the maximum compaction stress for repeatable behaviour. The model shows excellent agreement with results from compaction experiments on three materials, CFRM, CSM and Twill Weave.

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