ANALYSIS OF THE SATURATION IN LIQUID COMPOSITE MOLDING PROCESSES USING AN ESSENTIALLY NON-OSCILLATORY (ENO) TECHNIQUE

Ll. Gascón[1], J.A. García[1], F. Chinesta[2], E. Ruiz[3], F. Trochu[3]

[1] *Universidad Politécnica de Valencia. Camino Vera s/n, 46022, Valencia (Spain). Email: jugarcia@mcm.upv.es*

[2] *EADS Corporate Foundation International Chair, GEM – Centrale Nantes, 1 rue de la Noe, BP 92101, F-44321 Nantes cedex 3, France.*

[3] *Centre de recherche en plasturgie et composites (CREPEC), Département de Génie Mécanique, École Polytechnique de Montréal, Montréal, Canada, H3C 3A7.*

ABSTRACT: In the manufacturing of composite parts by Liquid Composite Molding process (LCM), complete saturation of the fibrous reinforcement is key. Incomplete saturation leads to voids within the fibers which cause failure of the final product. Thus, understanding of the formation of voids is necessary for proper molding. In order to analyze the formation of voids during the resin impregnation process, a one-dimensional solution based on two-phase flow through a porous medium, is proposed. This model leads to the introduction of relative permeability as a function of saturation and a modified equation for the saturation as a non-linear advection-diffusion equation with viscous and capillary phenomena, which depends on a number of factors. A detailed analysis is performed to assess the relative significance of the various input parameters on the saturation profiles. In order to numerically solve the modified saturation equation for the LCM process a high order essentially non-oscillatory (ENO) technique is proposed. The implemented algorithm allows a numerical optimization of the injected flow rate, which minimizes the micro/macroscopic void formation during mold filling. Some numerical results are presented and compared with the results taken from literature in order to validate the proposed mathematical model and the numerical scheme.

KEYWORDS: Liquid Composite Molding, Saturation, Void, Essentially Non-Oscillatory Techniques.

INTRODUCTION

Equations that describe the LCM filling process with void formation are based on a two phase flow model and lead to a coupled system of a nonlinear advection-diffusion equation for saturation and an elliptic equation for pressure and velocity [3]. In general, the saturation equation is a non-linear advection-diffusion equation which includes the capillary pressure effect and it reduces to a purely advection transport equation when capillary effects are neglected. The hyperbolic nature of the saturation equation and its strong coupling through relative permeability represent a challenging numerical issue. In this paper, a very accurate numerical approach is proposed to solve this complex flow behavior.

In previous works [1,2] different experiments were carried out to investigate the process of void formation. These experimental works shown that macrovoids tend to form during injection at low flow rates, due to capillary dominant effects, whereas high injection rates lead to microvoids formation. Based on these observations, a simplified mathematical model has been proposed in order to model the LCM filling process with void formation. In this model, the diffusivity coefficient in the saturation equation has been replaced by a term which depends on the velocity. The permeability is assumed to be a function of saturation, and then the continuity equation that governs the pressure distribution, includes a source term which depends on the saturation.

Essential to the optimum process design in LCM is the numerical simulation of the modified saturation equation. Many numerical methods to solve this type of equations suffer from serious nonphysical oscillations, excessive numerical dispersion or a combination of both. The technique here used for solving the advection-diffusion equation which governs the evolution of the degree of saturation of porous media is based on an essentially non-oscillatory fixed mesh strategy. For the ENO schemes, interpolation polynomials of one order less than the order of accuracy required in the solution are computed and these polynomials are a good approximation to the values of the numerical flux function at the cell walls. The key idea in the *r*th-order ENO schemes is to use the “smoothest” stencil among *r* possible candidates to approximate the fluxes at cell boundaries to high-order accuracy and at the same time to avoid spurious oscillations near shocks. However, they also have certain drawbacks. One problem is with the freely adaptive stencil choice, which could change.

GOVERNING EQUATIONS

The mathematical formulation of the saturation in LCM takes into account the interaction between resin and air as it occurs in a two phase flow. Combining equations that describe mass conservation and Darcy’s laws for resin and air phases as described in [3], the resulting equation for the saturation in its most general form gives

$$\phi\ \frac{\partial S}{\partial t} + \nabla\cdot\left(vf(S)\right) = -\ \nabla\cdot\left(D_{cf}(S)\nabla S\right) \tag{1}$$

where

$$f(S) = \frac{\lambda_R(S)}{\lambda_R(S)+\lambda_A(S)} \qquad , \qquad D_{cf}(S) = f(S)\lambda_A(S)\frac{\partial P_c}{\partial S} \tag{2}$$

Here $D_{cf}(S)$ is the nonlinear diffusivity coefficient due to capillary pressure P_c, defined as $P_c = P_A - P_R$; v is the total velocity, S is the degree of saturation of the reinforcement by the liquid resin, $\lambda_j(S) = K_{Rj}(S)\cdot K/\mu_j$ is the phase mobility, with $K_{Rj}(S)$ the relative permeability of the phase j, μ_j the viscosity of phase j and K the permeability tensor.

Replacing the total velocity v by $v_R + v_A$ and simplifying, Eqn. 1 can be rewritten as follows

$$\phi\ \frac{\partial S}{\partial t} + \nabla\cdot\left(v_R f(S)\right) = \ \nabla\cdot\left(\left(1-f(S)\right)\lambda_R(S)\frac{\partial P_R}{\partial S}\nabla S\right) \tag{3}$$

Assuming that the diffusivity coefficient depends on the resin flow velocity, as identified in experimental observations, leads to the following simplified equation,

$$\phi \frac{\partial S}{\partial t} + \nabla \cdot \left(v_R f(S) \right) = \nabla \cdot \left(\left(\alpha_m v_R + \frac{\alpha_M}{v_R} \right) \nabla S \right) \tag{4}$$

The term on the right side has been proposed in [1]. In this model, α_M and α_m represent the dispersive coefficients of the macro and microscopic voids, respectively.

Since the main interest is to simulate the flow of the resin phase, we can derive the governing equations for this problem combining, for the resin phase, Darcy's law (the subscript *R* for *v* and *p* has been omitted),

$$v = -\frac{K_R(S) K_{sat}}{\mu} \nabla p \tag{5}$$

with mass conservation

$$\nabla \cdot v = -\phi \frac{\partial S}{\partial t} \tag{6}$$

To derive a closed model, Eqn. 4 for the saturation has been considered. The simulation of the filling process involves the following operations at each time step:

1. Calculate the pressure distribution by applying a standard finite element discretization to Equation

$$\nabla \cdot \left(K_R(S) \nabla p \right) = \frac{\phi \mu}{K_{sat}} \frac{\partial S}{\partial t} \tag{7}$$

   where the relative permeability and the term on the right side depend on the saturation degree.
2. Calculate the velocity field from Darcy's law for the resin.
3. Update the saturation distribution by integrating Eqn. 4 using a fourth-order ENO technique (Algorithm ENO-4).

The boundary conditions are given by: the pressure gradient in the normal direction to the mold walls vanishes, the pressure or the flow rate is specified on the inflow boundary and the pressure is zero in the empty part of mold.

NUMERICAL SCHEME FOR THE SATURATION

In this study we describe a fourth-order essentially non-oscillatory scheme for the numerical saturation solution. The reconstruction algorithm is based on an adaptive selection of stencil for each cell so as to avoid spurious oscillations near discontinuities while achieving high order of accuracy away from them.

For the saturation equation

$$\phi \frac{\partial S}{\partial t} + \nabla \cdot \left(v \cdot f(S) \right) = \nabla \cdot \left(D \cdot \nabla S \right) \tag{8}$$

we define the flux as

$$F = \frac{1}{\phi} \left(v f(S) - D \nabla S \right) \tag{9}$$

and consider a uniformly spaced grid where each cell $I_j = [x_{j-1/2},\ x_{j+1/2}]$ has a width h. If Δt denotes the uniform time step, Eqn. 8 can be integrated by applying a conservative scheme

$$S_j^{n+1} = S_j^n - \frac{\Delta t}{h} \left(F_{j+1/2}^n - F_{j-1/2}^n \right) \tag{10}$$

We want to construct in each cell I_j a polynomial *p(x)* to evaluate at cell boundaries, such that

$$F_{j\pm\frac{1}{2}} = p\left(x_{j\pm\frac{1}{2}}\right) \tag{11}$$

Since the numerical flux function *F* is defined by the relation

$$\left(F_j\right)_x = \frac{F_{j+1/2} - F_{j-1/2}}{h} \tag{12}$$

we can observe that if *p(x)* satisfies

$$F(S(x)) = \frac{1}{h}\int_{x-\frac{h}{2}}^{x+\frac{h}{2}} p(y)dy \quad \Rightarrow \quad F_x = \frac{p\left(x+\frac{h}{2}\right) - p\left(x-\frac{h}{2}\right)}{h} \tag{13}$$

taking a derivative on both sides, which shows that *p(x)* is identical to the numerical flux function at the cell walls. Then we can calculate *p(x)* by finding its primitive

$$H(x) = \int_0^x p(y)dy \tag{14}$$

and then taking a derivative. Following the technique described in [4], the calculation of *H(x)* is based on polynomial interpolation. The main ingredient of the ENO method is the adaptive choice of stencil. For our case, it begins with a starting point to the left or right of the current cell by means of upwinding determined by the sign of the velocity; as the order of the undivided differences is increased, the undivided differences themselves determine the stencil: the smaller undivided difference is chosen from two possible choices at each stage, ensuring a smoothest fit. Then the following fourth-order algorithm based on the ENO-Roe numerical flux reconstruction can be described.

ENO-4 Algorithm for the saturation:

Step 1. Compute

$$v_{j+\frac{1}{2}} = \frac{1}{2}(v_j + v_{j+1}) \quad , \quad D_1F_j = F_j \quad , \quad D_2F_j = \frac{1}{2h}\left(F_{j+1} - F_j\right) \quad ,$$

$$D_3F_j = \frac{1}{6h^2}\left(F_{j+1} - 2F_j + F_{j-1}\right) \quad , \quad D_4F_j = \frac{1}{24h^3}\left(F_{j+2} - 3F_{j+1} + 3F_j - F_{j-1}\right)$$

Step 2.

If $v_{j+\frac{1}{2}} > 0$ then $k = j$, else $k = j+1$

If $\left|\mathrm{D}_2F_{k-1}\right| < \left|\mathrm{D}_2F_k\right|$ then $r = k-1$, else $r = k$

If $\left|\mathrm{D}_3F_r\right| < \left|\mathrm{D}_3F_{r+1}\right|$ then $s = r$, else $s = r+1$

If $\left|\mathrm{D}_4F_{s-1}\right| < \left|\mathrm{D}_4F_s\right|$ then $t = s-1$, else $t = s$

Step3. Compute

$$F_{j+\frac{1}{2}} = D_1F_k + h\cdot D_2F_r\cdot\left[2\left(j-k\right)+1\right] + h^2\cdot\, D_3F_s\cdot\left[3\left(j-r\right)^2-1\right] +$$

$$+ h^3\cdot\, D_4F_t\cdot\left[4\left(j-s\right)^3 + 6\left(j-s\right)^3 + 2(t-s) - 2\right]$$

In the numerical simulation of the modified saturation equation two terms contribute to smooth the flow front: one is related to the source term, and the other purely numerical term is introduced by the dicretization scheme. The last effect can be reduced by using a fourth-order numerical scheme.

Fig. 1 shows numerical results using the ENO-4 Algorithm for the saturation of the one-dimensional test described in [3]. A mold of length 1 m has been considered. The saturated permeability K_{sat} and the resin viscosity μ are set to 10^{-8} m^2 and 0.1 Pa.s, respectively. For the numerical simulation, we take $\alpha_M = 10^{-10}$, $\alpha_m = 10^{-3}$ and a constant injection rate of 0.001 m/s. The domain is assumed initially empty, except the first element that represents the injection nozzle that is assumed full-filled. Two different models for Eqn. 8 has been illustrated: $f(S) = S$, at the left, and $f(S) = S^2$, at the right.

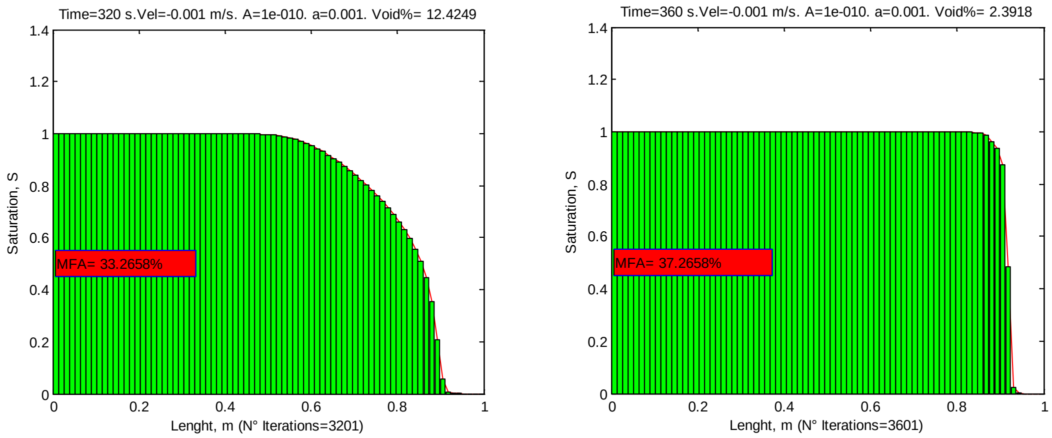

Fig. 1 Saturation profiles for a 1D RTM filling at constant flow rate [3]

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