# NUMERICAL SIMULATION OF COUPLED STOKES-DARCY FLOWS: APPLICATION TO LCM AT THE MESOSCOPIC SCALE

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**ABSTRACT**: In order to understand the different phenomena that occur during the forming phase in LCM (Liquid Composite Molding) processes, a multi-phase/multiphysics analysis must be undertaken. Three observation scales may be considered: macroscopic (process), mesoscopic (fluid/yarn), microscopic (fluid/fiber). At the macroscopic level, injection simulations require the determination of the permeability tensor. This tensor may be affected by the pre-forming step of the dry fabric. In this paper, a numerical study on the permeability determination is performed at the mesoscopic scale, on a 3D elementary cell, of the resin flow through this cell, when the fabric is considered has having been predeformed. A monolithic approach is coupled to an immersed volume technique: in a eulerian framework, the computational domain is composed of one single mesh, where the interface between yarns composing the deformed fabric and fluid is captured through a level set approach. Resolution of a coupled Stokes (in the fluid)-Brinkman (in the yarn) flow is necessary and is performed using a mixed finite element technique, providing a single system of equations. Results on elementary cells with fabric pre-deformation obtained also by simulation will illustrate the methodology followed.

**KEYWORDS**: permeability computation, monolithic approach, mixed finite elements.

## INTRODUCTION

LCM (Liquid Composite Molding) processes concern the injection of a resin matrix through a fibrous reinforcement placed in a complex shaped mold. As a consequence of the bad impregnation of the resin, one may encounter several problems, being the formation of porosities very critical for the part's mechanical properties. Simulation is used at the process scale to optimize the process, but does not predict the distribution of the porosity, since current macroscale models do not include the multi-scale nature of the fibrous media. At the macroscopic scale, one solves the Darcy equation by considering a homogeneous porous media. But in fact, fiber reinforcements are composed of several yarns that gather multiple fibers. Permeability used at the macroscopic scale may be more accurately determined by using direct simulation at the mesoscopic scale on REV (Representative Elementary Volumes), object of this paper. We will show that it may be computed by solving Brinkman's equations in the yarn and Stokes equations in the fluid, through an immersed volume technique. Numerical implementations and computations have been performed using CimLib, the scientific computation library developed at CEMEF, on which REM3D, a polymer and composite injection molding software is built from.

## IMMERSED VOLUME TECHNIQUE

We consider the computational domain composed of both the yarns and the resin. A multidomain problem is considered, with two phases, solid and fluid ant the problem resolution is done using a monolithic approach [1]: computation is performed using a single mesh that includes all the phases; interfaces between them are known implicitly through a distance function $\alpha$ to these interfaces (Figure 1).

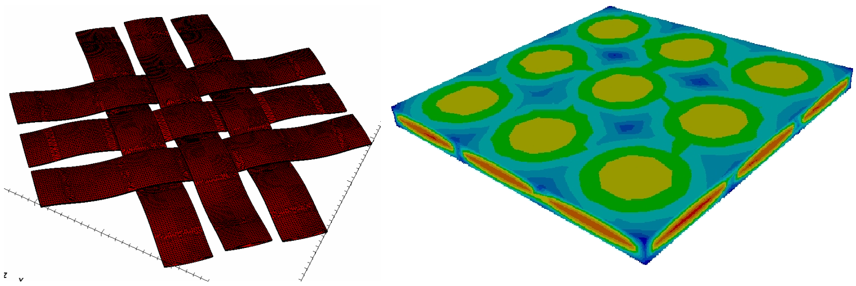

Fig. 1 Example of a computational domain at the meso scale. In the left; the yarn's meshes (courtesy of Lamcos-Insa de Lyon); on the right, the distribution of the distance function.

### Mixture law

Material properties, such as permeability or viscosity are computed on the whole computational domain through a linear mixture law:

$$\begin{aligned} \eta &= \eta_s H_\alpha(\alpha) + \eta_f [1 - H_\alpha(\alpha)] \\ \frac{\eta}{K} &= \left(\frac{\eta}{K}\right)_s H_\alpha(\alpha) + \left(\frac{\eta}{K}\right)_f [1 - H_\alpha(\alpha)] \end{aligned} \tag{1}$$

where $H_\alpha(\alpha)$ is the modified Heaviside function

$$H_\alpha(\alpha) = \begin{cases} 1 & if \quad \alpha > e \\ \frac{1}{2} + \frac{\alpha}{2e} & if \quad -e < |\alpha| < e \\ 0 & if \quad \alpha < -e \end{cases} \tag{2}$$

In these equations, $\eta$ is the viscosity, $\alpha$ is the distance function to the fiber-fluid interface (positive inside the yarn) and $e$ is the half-thickness of the mixture zone. The indexes $s$ and $f$ indicate the solid (yarn) and fluid (resin) domains, respectively.

## Mesh adaptation

A good accuracy on the description of the yarn-resin interface can be obtained through mesh control adaptation. Anisotropic mesh refinement at the interface level allows the control of the number of elements in the thickness e, as well as their orientation. To do that, an anisotropic metrics field is computed on the mesh, defining the mesh size in each spatial direction and is given to the mesher, incorporated in our solver. Being our interface defined by the gradients of a distance function, the metrics field better adapted is

$$M = m^2\left(\nabla\alpha \otimes \nabla\alpha^T\right) + \varepsilon^2 I \tag{3}$$

where $I$ is the identity tensor, and the mesh sizes in directions $\nabla\alpha$ and $\nabla\alpha^T$ are $1/\sqrt{(m^2|\alpha|^2+\varepsilon^2)}$ and $1/\varepsilon$. If we adapt the mesh only in the thickness $e$, the metrics field can be redefined as

$$M = \begin{cases} \varepsilon^2 I & if \quad |\alpha| > e \\ \left(\dfrac{N}{e} - \varepsilon\right)^2 \dfrac{\left(\nabla\alpha \otimes \nabla\alpha^T\right)}{|\nabla\alpha|^2} + \varepsilon^2 I & if \quad |\alpha| < e \end{cases} \tag{4}$$

where $N$ is the number of desired layers in the thickness $e$. In this thickness, we have $N$ elements of size $e/N$ in the direction $\nabla\alpha$ and the default mesh size $1/\varepsilon$ in the direction $\nabla\alpha^T$. Figure 2 illustrates adaptation at the yarn-resin interface.

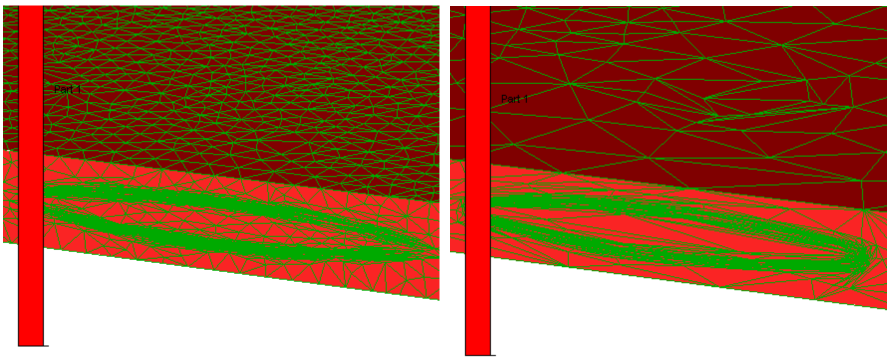

Fig. 2 Detail of anisotropic mesh adaptation at the yarn-resin interface. On the left, a 2.5 million element-mesh; on the right, a 500 thousand element-mesh.

## FINITE ELEMENT RESOLUTION

## Brinkman's equation

Darcy and Brinkman equations model flow through a porous media like flow through an equivalent continuous homogeneous media [2]. In the following, we suppose that the reinforcement is static and non deformable, the fluid is Newtonian, its density is constant and the media is saturated. Continuity equation, supposing that velocity at the pore surface is zero, is

$$\nabla \cdot v = 0 \tag{5}$$

where $v$ is the average velocity field. Averaging conservation of momentum leads to the Brinkman equation

$$-\frac{\phi\eta}{K}v + \eta\Delta v - \phi\nabla p = 0 \tag{6}$$

with $p$ the pressure, $K$ the permeability and $\phi$ the porosity. At the yarn-resin scale, we consider that $(\eta/K)_f=0$ in the resin and that there is no viscous term $\eta\Delta v=0$ in the yarn. Thus, we are led to the Stokes equations in the fluid domain and to the Darcy one in the solid. Through our mixture law, one can solve one single problem, Brinkman's, in the whole computational domain.

**Numerical resolution**

Brinkman's problem (8) is solved using a mixed finite element method and modified bubble stabilization, with a linear approximation in pressure and also a linear one in velocity. The enrichment performed stabilizes the formulation for both the Stokes and the Darcy cases, with a modification of bubble stabilization in the Darcy's region. The element contribution to the linear system arising can be written in the matrix form:

$$\begin{bmatrix} A_{vv} & A_{vb} & A_{vp} \\ A_{vb}^{T} & A_{bb} & A_{bp} \\ A_{vp}^{T} & A_{bp}^{T} & 0 \end{bmatrix} \cdot \begin{bmatrix} v_h \\ b_h \\ p_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

Bubble condensation provides a system in the main variables $v_h$ and $p_h$:

$$\begin{bmatrix} A_{vv} - A_{vb}A_{bb}^{-1}A_{vb}^{T} & A_{vp} - A_{1b}A_{bb}^{-1}A_{bp} \\ A_{vp}^{T} - A_{bp}^{T}A_{bb}^{-1}A_{vb}^{T} & -A_{bp}^{T}A_{bb}^{-1}A_{bp} \end{bmatrix} \cdot \begin{bmatrix} v_h \\ p_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{8}$$

where

$$\begin{bmatrix} -A_{vb}A_{bb}^{-1}A_{vb}^{T} & -A_{1b}A_{bb}^{-1}A_{bp} \\ -A_{bp}^{T}A_{bb}^{-1}A_{vb}^{T} & -A_{bp}^{T}A_{bb}^{-1}A_{bp} \end{bmatrix} \tag{9}$$

is the stabilisation matrix. The linear system resulting from the discrete formulation is solved using a conjugate residual method and an ILU preconditioning, using the PETSC library, to obtain the nodal distributions of the velocity and pressure fields.

## NUMERICAL RESULTS

First results concern the immersion of the yarns illustrated in Fig. 1, to compose a computational domain constituted of one ply reinforced matrix. The viscosity used was of 1 Pas and permeability of $10^{-12}$ $m^2$, for a fiber fraction of around 25%. A pressure gradient is imposed in the REV created and one may observe (Fig. 3) the distribution of the velocity vector in a cutting plane and in half the geometry of the REV. We can see that flow passes mainly in the inter-yarns' space, even if there is a very small velocity field inside the yarn.

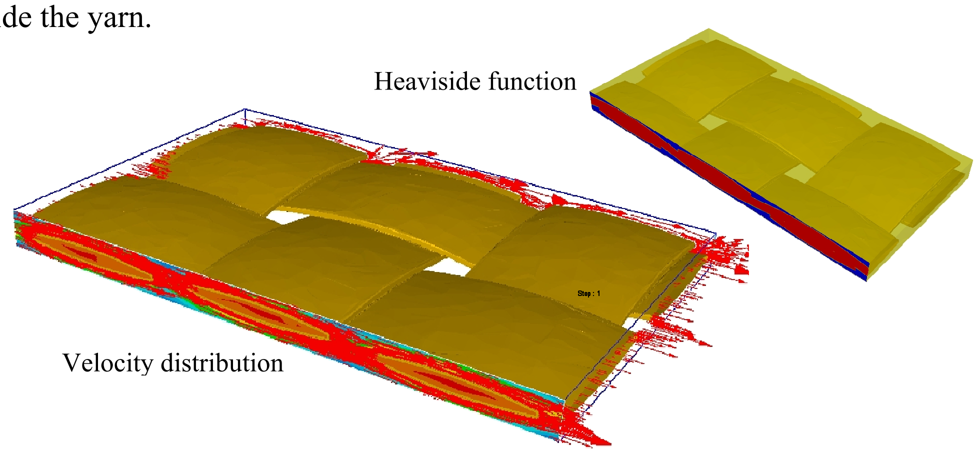


Fig. 3 Distribution of the velocity field on the half geometry defined through the immersion of one ply.

## CONCLUSIONS

To determine an accurate permeability tensor, a coupled Stokes-Brinkman problem must be considered (especially in what concerns transverse permeability prediction). A monolithic approach to treat the whole computational domain is presented in this paper. It is a promising way, because it allows taking into account the specific behavior of each phase, but it will also be possible to include in such a formulation a third domain, air, representing porosities. First results illustrate the methodology followed.

## ACKNOWLEDGEMENTS

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