PERMEABITY PREDICTIONS OF DUAL SCALE FABRICS USING LEVEL SET METHOD

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ABSTRACT: A new finite-element scheme which recasts the Stokes-Brinkman equation for flow analyses in dual-scale porous media as a single equivalent momentum equation over the entire domain has been developed and applied to predict the effective permeability of dual scale fabrics. The proposed scheme uses a structured regular rectangular mesh to discretize the domain and employs the level-set method to describe the porous media allowing for inclusion of complex geometrical features easily. Bi-periodic boundary conditions are applied for flow analysis in a representative volume of meso-scale porous structures. The scheme is applied to flow past two regular periodic geometries of elliptic fiber tows in 2D, representing uni-directional fiber tow in a textile fabric to predict the bulk or effective permeability and its dependence on the fiber volume fraction, the aspect ratio of the ellipse, the fiber tow permeability and the degree of compaction of the fiber tows. Only results of how the fiber tow permeability influences the effective permeability are presented due to space limitations.

KEYWORDS: Stokes-Brinkman equation, dual scale porous media, interfacial condition, level-set method, finite-element method.

INTRODUCTION

A new finite-element technique to solve the Stokes-Brinkman equations for the coupled flow problem in dual scale porous media, assuming that the inertia of a fluid between the fiber tows is negligible has been developed. Our scheme unlike previous efforts introduces a single equivalent momentum equation over the entire domain that accounts for both the continuous stress and stress jump conditions at the interface. The proposed scheme is constructed using the regular structured mesh and the porous media is described by the level-set method to incorporate complicated fabric structures easily, while attaining the smooth solution near the interface. Moreover, bi-periodic boundary conditions are implemented to solve for flow in the representative unit cell containing fiber tows efficiently. The overall bulk permeability relation to fiber tow permeability is explored and a constitutive form is suggested for two different fiber tow arrangements.

NUMERICAL METHODS

Fig.1 describes the coupled flow schematically. The computational domain is composed of the region occupied by the porous material and the region occupied by the surrounding homogeneous fluid. The interface between the two mediums is denoted by

Γ_{pf} and the normal vector on the interface is denoted by $\boldsymbol{n}$, which is the outward normal vector from the porous media and is opposite to normal from the fluid domain. The boundary of the computational domain is denoted by Γ_i and the horizontal (and vertical) periodicity in x (and y) is applied between Γ_1 and Γ_3 (Γ_2 and Γ_4). The pressure gradient is assigned either in the horizontal or in the vertical directions.

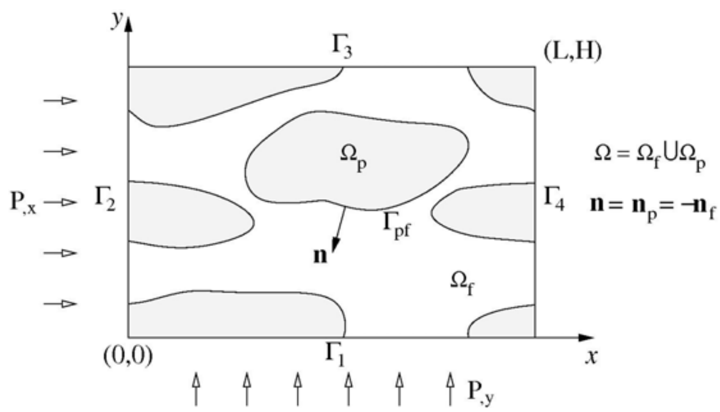

Fig. 1 A schematic description of the coupled flow formulation in a unit cell containing the porous medium. The subscript 'p' denotes the porous media and 'f' denotes the surrounding homogeneous fluid. Bi-periodic boundary conditions are applied and the pressure gradients in both directions can be taken into account

Using the Stokes theorem, one can express the single momentum equation that contains a distributed interfacial stress jump term such that the momentum equation is satisfied not only in the domain, including both fluid and porous, but also on the interface [1]. The momentum equation, called 'equivalent momentum equation,' can be written in rigorous form as follows:

$$\nabla \cdot \boldsymbol{\sigma}^* - \alpha \frac{\eta_f}{K_p} \boldsymbol{u}^* + \frac{\eta_f}{\sqrt{K_p}} \boldsymbol{T} \cdot \boldsymbol{u}^* \delta\left(\phi\left(\boldsymbol{x}\right)\right) = \mathbf{0}, \tag{1}$$

where the superscript '*' define the domain to which all physical quantities belong: in Ω_p, $\boldsymbol{u}^* = \boldsymbol{u}_p$, $\boldsymbol{\sigma}^* = \boldsymbol{\sigma}_p$, $p^* = p_p$ and $\eta^* = \eta'$; in Ω_f, $\boldsymbol{u}^* = \boldsymbol{u}_f$, $\boldsymbol{\sigma}^* = \boldsymbol{\sigma}_f$, $p^* = p_f$ and $\eta^* = \eta_f$. The variable α takes a value of '1' in Ω_p (porous domain) and '0' in Ω_f (fluid domain). For simplicity, we replace the second order tensor $\boldsymbol{T}$ by an isotropic tensor $\beta \boldsymbol{I}$ as was suggested by Ochoa-Tapia and Whitaker [2] for the one dimensional parallel channel flow and, if $\beta = 0$, the problem satisfies the continuous interface stress condition. The Delta function $\delta\left(\phi\left(\boldsymbol{x}\right)\right)$ with a properly defined level-set function $\phi\left(\boldsymbol{x}\right)$ is introduced in Eqn. 1 and the level-set function is a signed distance function from the interface: i.e., it yields a distance from the interface from a point $\boldsymbol{x}$ and is a negative (positive) value inside the porous (fluid) domain. The zero level set indicates the interface. Therefore, the Delta function gives an infinite peak on the interface and zero elsewhere. (The construction of the level-set function and the numerical 'smeared-out' Delta function is used for numerical implementation.) An example distribution of the level-set function for three elliptic tows in a bi-periodic domain is presented in Fig. 2.

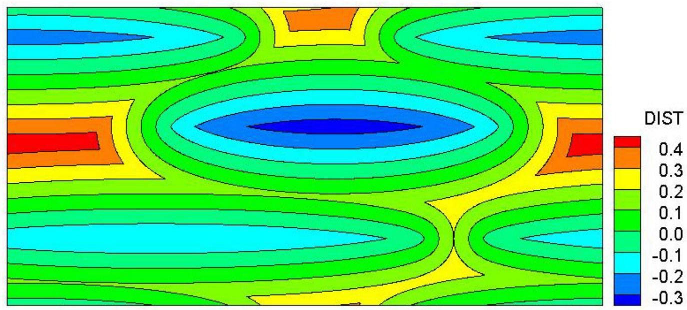

Fig. 2 An example distribution of the level-set function for three elliptic tows in a bi-periodic domain. Here zero distance is the interface, positive value signifies spaces between the tows while a negative value represents the fiber tow

NUMERICAL RESULTS

The examples explored are described in Fig. 3. We consider two fiber tow arrangements: one is a single elliptic tow, representing the porous media, at the center of the domain ($L = 6.6$ [mm] and $H = 1$ [mm]); the other is a two-elliptic-tow flow in a larger domain ($L = 8$ [mm] and $H = 1.65$ [mm]), where a tow is located at the center and the other tow is located at the corner (split into four sections). Due to the bi-periodicity, they represent a regular stack and a squeezed hexagonal arrangement of tows, respectively, in an unbounded domain.

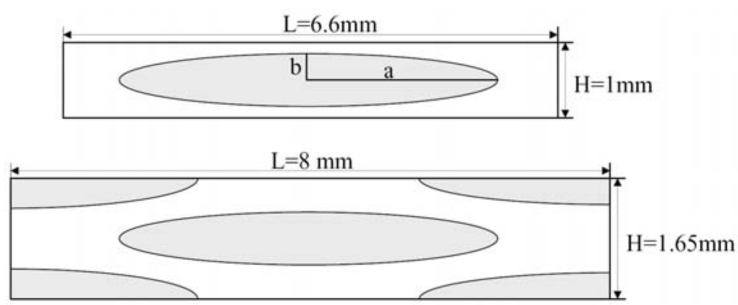

Fig. 3 A single-tow and two-tow arrangement in a bi-periodic domain. The tow volume fraction is the same for both cases.

We have investigated the effective or bulk permeability, (K_{xx} and K_{yy} in the x and y direction respectively) and its dependence on the fiber volume fraction, the aspect ratio, the fiber tow permeability and the degree of compaction of the fiber tows. Due to the page limitation, we present only one result: influence of tow permeability on the effective permeability of the fabric. The results are presented in Fig. 4. Even with 10^4 times increase in the tow permeability there is not much change in the effective permeability: from the minimum of about 1.8 times increase in K_{yy} in the single-tow problem to the maximum of around 16 times increase in K_{yy} in the two-tow problems. Such a minor dependence originates largely from the magnitude of the Darcy velocity u_d within the tow, which is much smaller than the magnitude of the pressure driven flow velocity. In fact, one can model a simple relationship on the dependence on K_p. Among various contributions on the velocity, the slip velocity, which is most dominant in case of $\sqrt{K_p} \gg K_p$, scales with $\sqrt{K_p}$. Another important contribution is the pressure driven flow, a flow with the impermeable wall instead of the permeable porous media, and it is independent of K_p. From this understanding we propose the following simple relationship between the effective permeability K and the tow permeability K_p as long as $\sqrt{K_p} \gg K_p$:

$$K \approx C_{pressure} + C_{slip}\sqrt{K_p},$$

where $C_{pressure}$ and C_{slip} represent the pressure flow contribution and the coefficient of the slip flow rate, respectively. We carried out the least square fitting for the data given in Figure 4 as well.

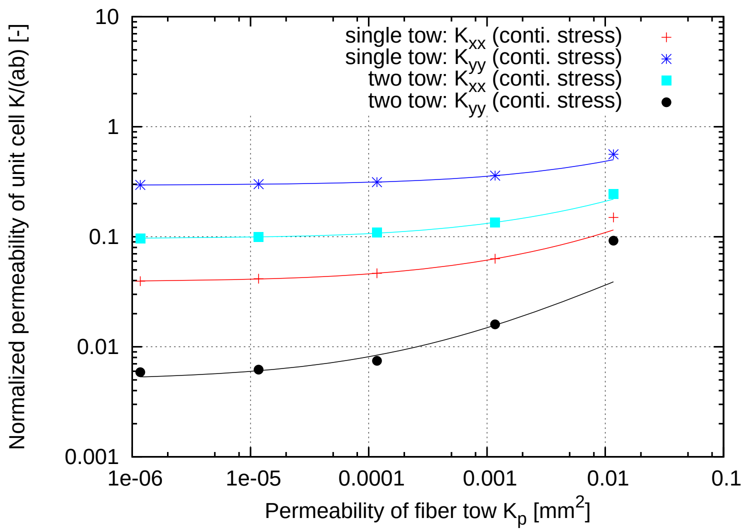

Fig. 4 The effect of the fiber tow permeability on the effective normalized permeability (with respect to the fiber tow area).

CONCLUSIONS

A new finite element scheme to solve the Stokes-Brinkman equation for flow analyses in dual-scale porous media has been presented and applied to predict the effective permeability of dual scale fibrous porous media. Introducing the equivalent momentum equation, we transformed the stress jump conditions at a sharp interface into an additional distributed body force near the interface within a single momentum equation over the entire domain. The method is quite suitable for a numerical method based on a regular mesh. We employed a level-set method to describe the geometric arrangement of porous media efficiently and introduced the diffuse interfacial body force in a consistent way. In addition, we demonstrated this flow analyses in a representative unit cell of meso-scale dual scale fabrics containing elliptical tows, employing the bi-periodic boundary condition.

The combination of this method with a direct simulation technique used to predict the permeability of a fiber tow, e.g., Wang and Hwang [3], may yield purely computational permeability prediction tow to explore permeability of complicated fabric architectures with dual scale pores. This flow solver can be employed as a base flow solver for analyses of various physical and industrial problems in composites processing with the resin infusion. For example, when particle fillers are blended with resins to achieve a specific property, particle filtration and deposition due to micro- or meso-scale porous structures might be reasonably predicted combining this solver with a particle concentration evolution equation or a direct particle suspension solvers [4,5]. Another interesting future application is in orientation prediction of carbon nanotubes (CNT) in such a process. One can combine this solver with statistical orientation evolution equations of CNT using either Fokker-Plank or Langevin-type kinetic models [6].

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