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INFLUENCE OF STOCHASTIC FIBRE DISTRIBUTION ON PERMEABILITIES OF FIBRE BUNDLES

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SUMMARY: In spite of numerous objections concerning the validity of Darcy's law, it is widely applied in simulation of manufacturing processes of composite materials due to its simplicity. The permeability of fibre network was found to be dependent on parameters such as fibre volume fraction and fibre diameter, etc. However, many factors influencing permeability have gained little or no attention in literature, such as fibre compaction by applied pressure and stochastic characteristics of the fibre network. In this work, Morishita's algorithm for characterising stochastic fibre arrangements was applied to cross sections of actual fibre bundles. The stochastic characterisation requires a few parameters, which can be determined by optical methods. It was found that a quantity of about 100 fibres is necessary to obtain representative values for a characterisation. A fluid simulation process was developed to define the permeability. A region of 15 fibres, artificially arranged in order to obtain the same distribution characteristics as the real fibre bundle, was found to be sufficient to determine the transverse micro-permeability of a complete fibre bundle. The simulated permeabilities correspond very well with results found in literature.

KEYWORDS: Transverse Permeability, Characterisation, Stochastic Fibre Distribution, Morishita's Algorithm, CFD, Micro Geometry.

INTRODUCTION

For calculating the necessary parameters of impregnation processes of fibre reinforcement during composite manufacturing by liquid state matrices, Darcy's law has gained wide application:

$$\dot{q}_z = -\frac{K}{\eta}\frac{dp}{dz} \tag{1}$$

$\dot{q}_z$: Penetration rate p: Pressure

K: Permeability η: Viscosity
z: Direction of the co-ordinate

Numerous objections exist against the application of Darcy's law, e.g. the permeability is an empirical value to be determined by experiments and the approach was deduced for newtonian media, which does not apply to most of the flow processes in composite materials. Therefore, in the past, many efforts have been made to refine the approach. The Carman-Kozeny equation was also widely used, which expresses the permeability as a function of fibre volume content and fibre diametres:

$$K = \frac{d^2 (1 - \xi_F)^3}{16 K_{CK} \xi_F^2} \tag{2}$$

d: Mean fibre diameter K_{CK}: Carman-Kozeny constant ξ_F: Fibre volume content

The Carman-Kozeny constant should be independent on fibre volume content and fibre diameter. A remarkable number of literature about experimental and theoretical determination of K_{CK} exist, and an overview can be found in [1]. For common fibre volume contents between 0.3 and 0.8, its value is about 5.0 [1,2]. It was shown by transverse impregnation experiments [3] that K_{CK} strongly depends on the applied pressure and the matrix' shear thinning behaviour. The values vary between 6 and 8. Most relevant literature deals with fibre networks with regular fibre arrangement. Comparing experimental results and calculations of the Kozeny constant in case of regularly arranged fibres, it was found that the estimated constant is too high [1]. Therefore, it can be stated that the micro-geometrical fibre distribution must have a decisive influence on the permeability.

PERMEABILITY AND STOCHASTICS

Only a few researchers considered stochastic effects on flow permeability [2]. Comparisons of dimensionless permeabilities dependent on porosity have shown a noticeable variation [3-4], thus leading to the assumption that the permeability seems to depend strongly on the fibre distribution [2].

Before the influence of the statistical fibre distribution on the permeability can be determined, the distribution needs to be characterised. This was done in many of the above mentioned publications [2, 4-9]. However, many of those methods are not very useful for practical applications because some of the characterisation parameters are difficult to handle. Some of these parameters were "minimum distance of a fibre *i* to the next surface" [2], "mean fibre number on any point ***y***", "covouring (mean surface of a fibre with contact to another fibre, related to total surface of fibre)" [6], "probability that a molecule touched a fibre at position ***y*** hits on a direct way another fibre at the position ***y***'" [8], "mean slit width between two neighbouring fibres" [9], and "volume related total size of fibre surface" [7], etc. Characterising a stochastic distribution with only a few parameters is a highly nontrivial task [4].

The finite element method is a very powerful method to numerically simulate nearly arbitrarily difficult geometries and material behaviour. Therefore, a couple of stochastic problems can be resolved using this method. Analytical results on permeability of porous rocks were correlated with a simple FE model [10], experimental results [11]. In the approach, the local permeability was set by a Monte Carlo algorithm and the behaviour of the flow was simulated by a finite difference method. A model with stochastically arranged fibres was developed in [2], and the permeability of a saturated flow was calculated. At all 4 borders of a 2D quadratic flow field, periodic boundary conditions were used and a so-called super cell was created. The issue of contacting fibres was solved by using two models. One was made without contacting fibres for an upper limit for the permeability, and the other was with contacting fibres for a lower limit for the permeability. It was shown that at fibre volume contents of more than 0.6, the 2D model became unprecise, because many fibres blocked the flow path, which does not appear in reality, because the fluid can always elude in the third dimensional direction. The works dealing with stochastic fibre arrangements leave many questions unanswered [2]. The answers to some of those questions are motivation of this work.

CHARACTERISATION OF STOCHASTIC FIBRE DISTRIBUTION IN FIBRE BEDS

The main objective of stochastic characterisation of fibre distributions is to determine the minimum number of fibres to be evaluated in order to obtain a reliable characterisation of the whole fibre bundle. The stochastic distribution of the fibres in the bundle cross section was determined as described in [12] by Morishita's quadratic method [13]. It was necessary to section a specimen perpendicular to the fibre bundles and to generate a list of all fibre positions by an image processing code, in order to get the necessary input data for the characterisation algorithm (more details can be found in [14]).

Morishita's index is calculated as follows:

$$I_\delta(q) = q\delta(q) \tag{3}$$

$$\delta(q) = \sum_{i=1}^{q} n_i(n_i - 1) / N(N-1) \tag{4}$$

N: Total number of fibres; n_i: Number of fibres in sub section i

q: Number of sub sections

The area under consideration is divided into a number q of so-called sub-regions with same shapes and sizes. Useful values for q are therefore 1 and the square numbers 4, 9, and 16 etc. Then, a diagram can be created with I_δ depending on q. The shape of the curve enables a quick overview on the distribution characteristics [13].

Morishita's algorithm was applied to actual fibre distributions. For doing this, by the aid of the film stacking technique, specimens were produced in a heating press by stacking of PA6.6 films of a thickness of 50 µm (Zytel 101, DuPont) and glass fibre plain woven fabric (7581/A1100, Hexcel) at a temperature of 275 °C and at different pressures and holding times. Specimen geometry was 100 mm x 80 mm x 1 mm. Cross sections were prepared at the centre of each specimen. Observed under an optical microscope, the fibre bundles have elliptical shapes in the cross section, and an example can be seen in Fig. 1.

In the ellipse, a central region with about 100 fibres was chosen and the fibre positions were evaluated by Morishita's algorithm. I_δ possesses a value of 0.44 at $q = N$. This means that the characteristic of the fibre distribution is not purely stochastic [12], but possesses a certain regularity. The examined geometry is amplified in Figure 2.

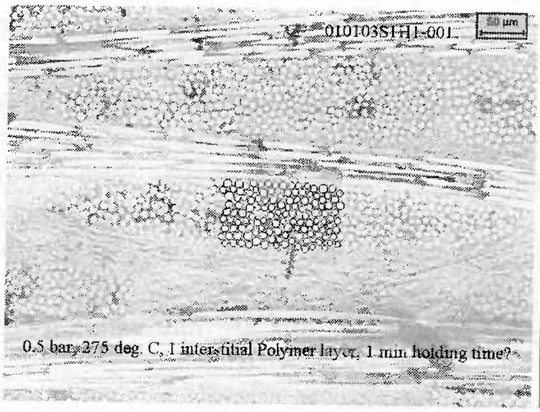

Figure 1 Fibre distribution in a fibre bundle in a GF/PA6.6 composite

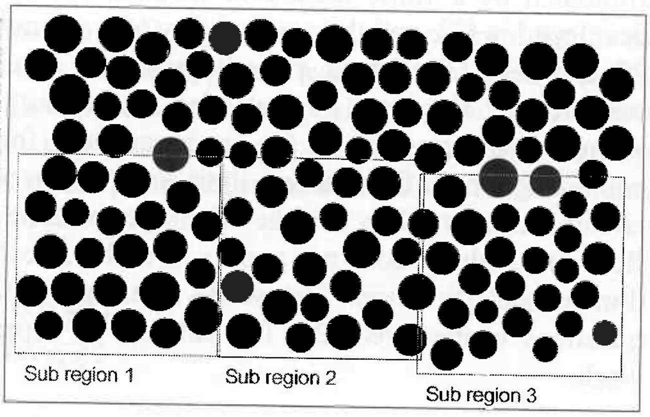

Figure 2 Magnified section of sub-regions under consideration

As a next step, it should clarify what is the region of minimum number of fibres to obtain the same Morishita's number of 0.44 as in the complete region. Three smaller sections (sub regions) were chosen out of the total region in Fig. 2. The evaluation of the sub regions with each about 25 fibres leads to $I_\delta (q)$ of 0.21, 0.30 and 0.36, far below 0.44. Therefore, it can be stated that an evaluation of only 25 fibres is not sufficient to obtain a Morishita's number representing the whole fibre bundle ellipse. The evaluation of at least 100 fibres is necessary to obtain the distribution characteristics of the fibre network of a fibre bundle.

Some more detailed work on this subject can be found in [12], where some results were also presented for the influence of the applied pressure on the fibre distribution characteristics. It was found that an increase in pressure leads to a higher Morishita's index, this means, a lower degree of regularity in the fibre arrangement.

FLOW SIMULATION OF CHARACTERISTIC FIBRE DISTRIBUTIONS

The aim of the flow simulation is to systematically and comprehensively examine the influences of geometrical parameters of the fibre bed (here especially the characteristics of the stochastic fibre distribution) on the permeability. The mesh of the 2-dimensional region was created with PATRAN. FIDAP (Fluent Inc., Lebanon, NH, USA) was used to especially simulate the fluid dynamics of flow processes. Initial parameters are the inlet speed and the data of the fluid. Apart from the local flow speed in the fibre bed, FIDAP presents the pressure drop of the fluid and the corresponding flow distance of the fluid. Based on these data, the permeability of the fibre bed can be calculated.

Applied method

Some peculiarities of the mesh generation and conversion can be found elsewhere [14]. However, some details about the unit system are explained in the following.

Using the normal kg-m-s (SI) unit system would have lead to severe convergence problems of the simulation due to considerable numerical rounding errors. Some variables with their typical orders of magnitude are shown in Table 1. The minor dimension of a typical flow region is L = 100 µm, and a typical impregnation time is t = 100 s, leading to a typical flow speed of v = 1 µm/s. The PA6.6 matrix has a density of ρ = 1140 kg/m^3 and a viscosity of η = 65 Pa.s.

From columns 3 and 4 in Table 1, it can be seen that the numerical values vary between 10^6 and 10^{-6} in the SI unit system, but only between 10^3 and 10^{-4} in the 10kg-cm-s – system. An alternative would have been a dimensionless approach, however the low Reynolds number of only 1.75 x 10^{-9} also was expected to create numerical problems.

Table 1 Conversion of physical parameters into the 10kg – cm – s system

Parameter	Typical value	Kg – m - s	10kg – cm - s
L	100 µm	10^{-4} m	10^{-2} cm
p	1 MPa	10^6 kgs^{-2}m^{-1}	10^3 (10kg)s^{-2}cm^{-1}
t	100 s	10^2 s	10^2 s
v	1 µm/s	10^{-6} ms^{-1}	10^{-4} cms^{-1}
ρ	1140 kg/m^3	1.14·10^3 kgm^{-3}	1.14·10^{-4} (10kg)cm^{-3}
η	65 Pas	6.5·10^1kgm^{-1}s^{-1}	6.5·10^{-2} (10kg)cm^{-1}s^{-1}

The boundary conditions are shown in Figure 3. An initially filled region, tagged light grey, presents at the left hand side. Inside this region, the velocity component in the y direction is set to zero. The matrix enters into the larger, initially empty region with a speed of 1 µm/s. At the surface of the fibres both velocity components are zero. At the top and bottom boundaries, the initially applied continuous boundary conditions caused some difficulties with the software used. Therefore, u_y was set to 0, forcing the field to be a kind of mirror-symmetric.

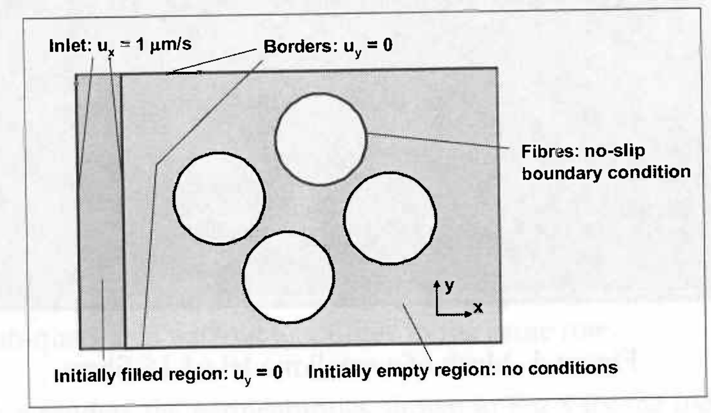

Figure 3 Initial and boundary conditions of a simple model

Some efforts were made to accelerate the simulation process. Amongst others, a quasi-Newton solver [15] was applied, which was much faster as the original applied segregated solver. A backward step iteration scheme was used, the time step was fixed to 0.1 s at the beginning; it was then reduced by FIDAP, depending on the local gradients of pressure and velocity and the local element size. The efforts lead to a reduction of 90 % in CPU time.

First results

A model of a fibre area of 25 fibres with the Morishita's index $I_\delta\,(q=N=25)$ = 0.45, length 61.6 µm, was first evaluated [12]. The fibres were artificially arranged in such a manner, that the same Morishita's index as in the real fibre bundle (determined by evaluation of the positions of 100 fibres) was retained. Result was a permeability of $1.8^{-13}m^2$, which corresponds well to data found in literature [16, 17], if the value for the fibre volume fraction is extrapolated. Meanwhile, after a flow time of only 30 s, the value for the permeability approached constant until the model was finally filled after about 60s [12]. This indicates that a smaller model could lead to the same result with a reduced CPU. Therefore, a smaller model with 15 fibres was developed in the further evaluation.

Minimum model size

For a constant Morishita value $I_\delta\,(q=N)$ = 0.45, the model with a relevant model length of 35.5 µm, containing 15 fibres (N=15), was constructed, and the corresponding mesh can be seen in Fig. 4. The model possesses 5127 nodes and 5995 elements. The distortion value was above 0.3 for all elements and above 0.5 for more than 99 % of all elements. The aspect ratio was above 0.3 for all elements and above 0.4 for 99 % of all elements.

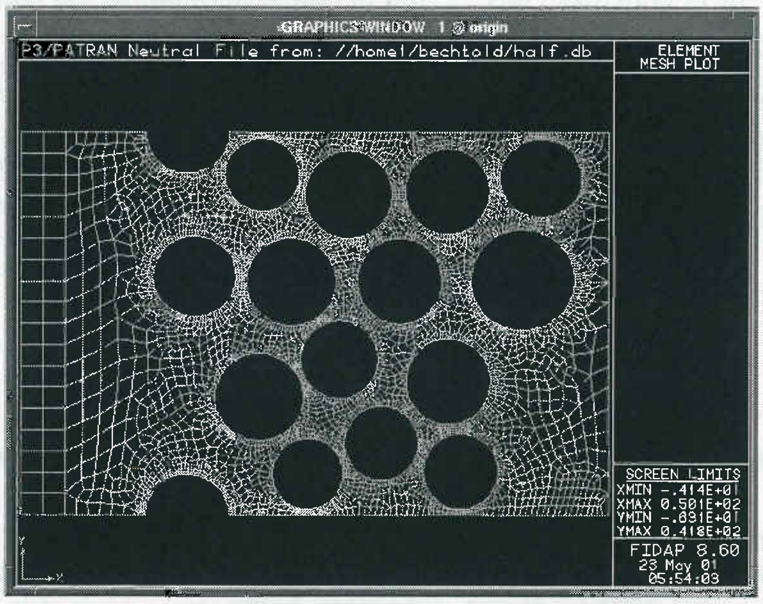

Figure 4 Mesh of a small model of 15 fibres

Initial and boundary conditions as well as the material properties were the same as those for the large model. The CPU time was about 21 h. Fig. 5 shows the behaviour of the fluid after 28.4 s.

The result confirms that the permeabilities determined by both models (25 and 15 fibres) were both about $1.8^{-13}m^2$. This indicates that the small model leads to the same result with a simpler model and a much smaller amount of CPU (21 h for 15 fibres compared to 41 h for 25 fibres). Therefore, in future work, we will limit the models to 15 fibres.

Correlation Morishita's number and permeability

It is interesting to show if and how the behaviour of the Morishita's index depending on q influences the permeability of the flow field. The model with 15 fibres was used as a basis, and its distribution can be seen in Fig. 6 with the label "A". The same fibres, however with different positions, were used for all other three distribution patterns. Also the size of the field was the same in all cases. In the 16 sub quadrates of every distribution, determined by grey lines in Fig. 6, 3 sub-quadrates were assigned by 2 fibres, 4 with no fibres and the rest with 1 fibre each, indicated by grey numbers at the centre of the sub-quadrates. Therefore, in each case, at a quadrate number of 16, the Morishita's index of 0.46 was achieved, see Fig. 7.

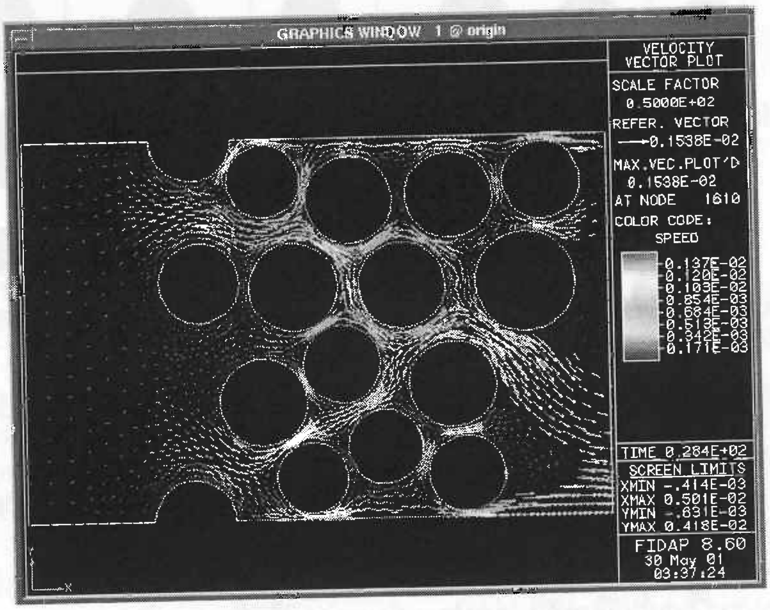

Figure 5 Matrix flow after 28.4 s

On top of Fig. 7 is shown the configurations of sub-regions and the number of quadrates q. Based on the configurations, the relationship between I_δ and q for the fibre distribution patterns is established. It is interesting to see that the curves for models A and D are absolutely identical. Quite similar to both is model E; however, model B shows a different behaviour. At a quadrate number q of 4, the value of the Morishita's index of B exceeds 1, and not before $q = 36$, an I_δ of 0 is reached, whereas all the other curves remain below 1 and reaches 0 at a quadrate number q of 25. This can be explained by the large flow channel in model B of four sub-quadrates without any fibres in the same row.

The CFD simulation renders the permeabilities shown in Fig 8 for the individual models of 15 fibres. The permeabilities drop at the first few seconds of the flow process and then approach a constant value after about 20 to 25 s. It can be clearly seen that the models A, D and E approach the value of about 1.8^{-13}m^2. Model B however approaches very fast its considerably high constant value of $1 \cdot 10^{-12}\text{m}^2$.

From these results, it can be stated that a simple characterisation by $I_\delta(q=N)$ is not sufficient to obtain correlations with the permeability, and the behaviour of the whole Morishita index curve, $I_\delta\ (q)$, influences the permeability. At least the smallest quadrate number for Morishita's index to approach to 0 ($q_{min}(I_\delta=0)$) must be added to the characterisation parameters.

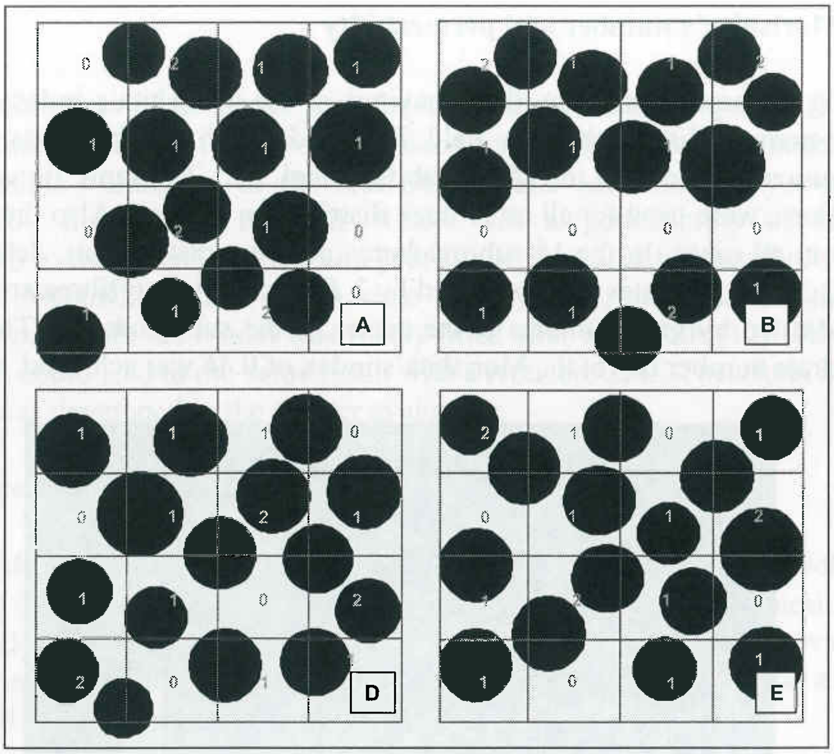

Figure 6 Four fibre distribution patterns with same Morishita's index of 0.46

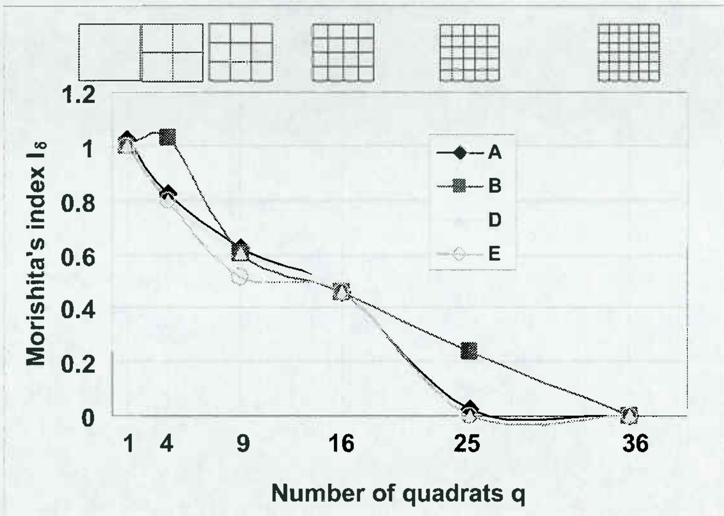

Figure 7 Morishita curve for presented fibre distribution models

CONCLUSIONS

Evaluating a region with about 100 fibres is sufficient to characterise a whole fibre bundle by the Morishita square method. A mesh generation of a 2-dimensional region with stochastic fibre arrangement by PATRAN and a following transient fluid flow simulation by FIDAP for prediction of permeabilities were conducted with a limited amount of CPU. A 15 fibres region is sufficient to obtain realistic values for the permeability. Unfortunately, a characterisation of the stochastic fibre distribution by only one value of Morishita's index is not sufficient. However, the distributions with similar curves of Morishita's index lead to similar permeabilities.

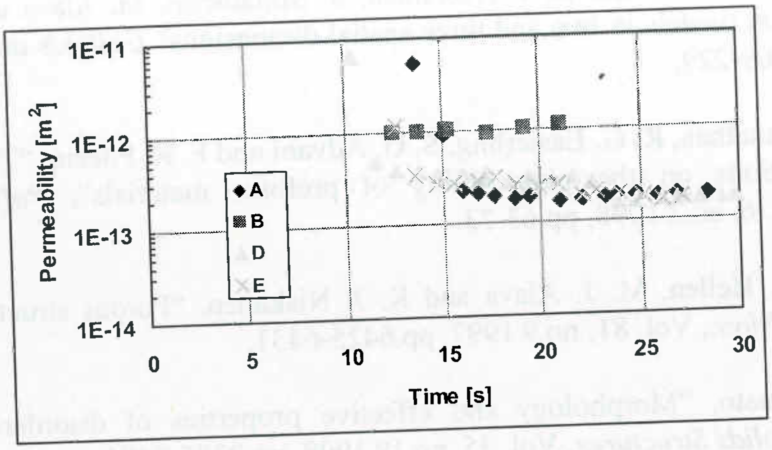

Figure 8 Permeabilities of presented fibre distribution models

FUTURE WORK

Apart from the presently used GF/PA6.6 material, other fibre/matrix combinations will be characterised by sectioning and application of the Morishita algorithm. New experimental results to determine the permeability would supplement the comparison of the simulated results with those found in literature. The influence of the pressure on the fibre bed's permeability through its influence on Morishita's number and the local fibre content will be determined by a FE model.

A further extension of the present method could be the examination of the influence of other parameters on the permeability, namely viscosity, global fibre volume content, fibre diameter and its standard deviation, surface tension and visco-elasticity.

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