Numerical Simulation of Mould Filling for Three-Dimensional Composite Parts

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Abstract – **The object of this paper is to present a finite element procedure to simulate mould filling in the three dimensional case. The method presented in the paper is div-conform i.e. the mass of injected fluid is proven to be perfectly conserved. For 2-D problem, classical finite element techniques give fine results event without ensuring the conservativity of the numerical scheme. In 3-D, we show that classical finite element approximation give such poor results that they cannot be considered as acceptable. With the new method, we obtain good quality results event with coarse meshes.**

INTRODUCTION

Until now, mostly thin shell parts were manufactured by Liquid Composite Molding (LCM). In such parts, the part is a considered as a two-dimensional manifold (a surface). The resin flow in the reinforcement is considered to be tangent to the manifold so that no transversal flow exists. For such kind of configuration, it has been proved that numerical simulation gives accurate and reliable results for predicting meaningful industrial parameters such as filling time, front position or maximal pressure[1].

Nowadays, manufacturers want to use the LCM process family for thick parts when the shell assumption cannot be considered as valid. The present paper is concerned with the numerical simulation of three dimensional mould filling in thick composite parts.

Considering that three dimensional models are just an extension to two dimensional ones is a simplistic point of view. It is true that mathematical models such as Darcy's equation for flow in porous media are not dependant on the dimension of the manifold. Similarly, numerical methods for solving the problem such as the finite element method have the same mathematical kernel for 2-D and 3-D. The principal problem in 3-D is to obtain accurate results using sufficiently coarse meshes. In the paper, we will demonstrate that classical finite element techniques are not suitable for simulating LCM processes with reasonable meshes i.e. meshes for which we are able to complete the filling of a part in less than two hours on a small P.C. For such kind of meshes, the classical conform finite element interpolation gives a loss of resin flow rate of order of magnitude of 50 %. This means that, when two kilograms of resin are injected, one is lost or created in the numerical process. This is of course not acceptable. These problems occurs also in two dimensional simulations but with a very lower amplitude.

The new approach consists in finding a new finite element approximation that have the property to ensure a conservative solution. The flow rate is proved to be conserved, even for very coarse meshes.

Examples of simulation are presented which incorporate different pressure boundary conditions at the injection gate and on the resin front. The numerical algorithm is proved to be able to predict accurately mould filling for three dimensional configurations.

1 Mathematical model

Classically, the problem of modeling the cavity filling by a fluid is divided in two parts (Fig. 1).

First, one must know the velocity distribution of the fluid in the cavity: this constitutes the flow model. Note that the fluid is present on a given part of the cavity delimited by the walls, the injection ports and the front flow.

When the velocity of the fluid is known, one must perform the advancement of the front flow. This model is simply the one of a pure transport equation.

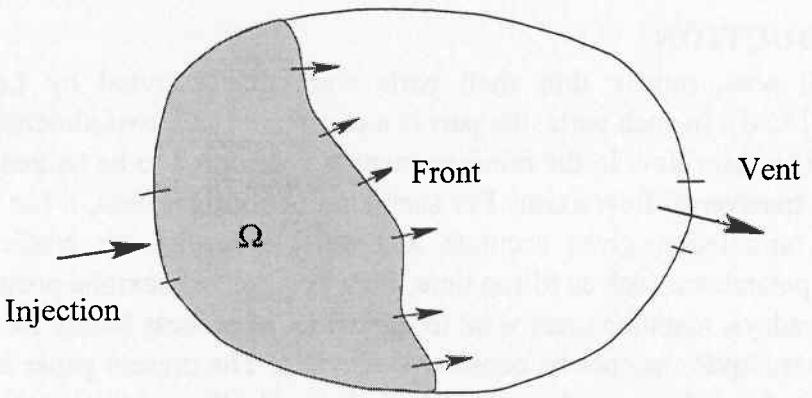

Fig. 1: Configuration for mould filling

1.1 Flow model

The motion of a Newtonian fluid through a porous medium may be predicted by *Darcy*'s law:

$$\mathbf{v} = -\frac{K}{\mu}\,\mathrm{grad}\; p, \tag{1}$$

where p denotes the pressure field, K the permeability tensor and μ the resin viscosity. The permeability characterizes the porous material in terms of resistance to the fluid flow for a given injection pressure. The *Resin Transfer Molding* (R.T.M.) process consists of injecting at low pressure a thermosetting resin into a mold cavity filled with a dry fibrous preform. When the mold is completely filled, the part is cured and

subsequently demolded. We assume that the resin flow through a porous medium obeys to the *Darcy* equation.

The injected resin is considered as incompressible i.e. the density ρ of the resin is not modified by the flow. This can be expressed mathematically by writing that the Lie derivative of ρ in the flow **v** is equal to zero [2] :

$$L_v \rho = 0$$

which is equivalent to :

$$\operatorname{div}(\rho \mathbf{v}) = 0 . \qquad (2)$$

Equations (1) and (2) form a system of elliptic partial differential equations that can be easily written in a weak form. Considering a closed domain Ω of IR^3 and its boundary Γ composed of two complementary parts Γ_v and Γ_p and using standard notations $(\cdot,\cdot)_\Omega$ and $<\cdot,\cdot>_\Gamma$ for volume and surface integrals are defined respectively as :

$$(\boldsymbol{a},\boldsymbol{b})_\Omega \overset{def}{=} \int_\Omega \boldsymbol{a}\cdot\boldsymbol{b}\, dv, \quad \langle a,b\rangle_\Gamma \overset{def}{=} \int_\Gamma ab\, ds ,$$

a weak formulation of the problem (1-2) consists of finding p in $H_0^1(\Omega)$ solution of :

$$(-\mathrm{K}\ \mathrm{grad}\ p, \mathrm{grad}\ p')_\Omega - \langle (\mathbf{n}\cdot\mathbf{v}), p'\rangle_{\Gamma_v} = 0 , \quad \forall p' \in H_0^1(\Omega) \qquad (3)$$

with the classical Sobolev function spaces:

$$\mathrm{H}^1(\Omega) = \left\{ \mathrm{v} \in \mathrm{L}^2(\Omega); \frac{\partial \mathrm{v}}{\partial \mathrm{x}_i} \in \mathrm{L}^2(\Omega); 1 \le \mathrm{i} \le 3 \right\} \text{ and } \mathrm{H}_0^1(\Omega) = \left\{ v \in H^1(\Omega); v|_{\Gamma_v} = 0 \right\}.$$

1.2 Front advancement Model

Classically, we define in Ω a scalar field f which value is equal to one when the resin is present (saturated medium) and is equal to zero when it is not (wet medium). This scalar represents the concentration of the resin in the porous media. This concentration is one at injection ports. Initially, all the medium is wet. Then, the concentration on the injection ports is transported in the flow. This can be expressed by the pure transport equation:

$$Df = \frac{\partial f}{\partial t} + L_v f = 0$$

which becomes here :

$$\frac{\partial f}{\partial t} + v \cdot \mathrm{grad}\ f = 0 .$$

Note that if the fluid is not Newtonian, one must also transport its shear stress tensor to be able to predict its viscosity. If the fluid is reactive, its chemical properties have to be transported. Finally, if the problem is not isothermal, one must also transport the temperature of particles. The Lie derivative relative to **v** L_v gives a simple way to

predict the variation of a object (scalar, tensor, density) in a flow. For scalars (e.g. temperature T), we have $L_v T = \mathbf{v}\,\mathrm{grad}\,T$, for densities (mass density ρ), we have $L_v \rho = \mathrm{div}(\rho \mathbf{v})$.

2 Numerical Methods

2.1 Finite elements

The finite element method is characterized by a double discretization. The geometrical domain Ω is divided in simple geometrical elements (the mesh is this geometrical discretization) and the continuous function space (e.g. $H_0^1(\Omega)$) is discretized such that the approximation space for the unknown is of a finite dimension (shape functions for functional discretization). It is this double discretization (the mesh and the shape functions) that entirely determinates the approximation space V_h.

One often choose to associate one approximation function s_n for every node of the mesh. The pressure field p is approximated as :

$$p(\mathbf{x}) = \sum_{n \in N} p_n s_n(\mathbf{x})$$

Problem (3) is discretized using finite elements. We have to find p in V_n such that :

$$\sum_{n \in N_v} \left[p_n \left(\mathrm{K\ grad}\ s_n, \mathrm{grad}\ s_m \right)_\Omega \right] = \left\langle \mathbf{n} \cdot \mathbf{v}, s_m \right\rangle_{\Gamma_v} \quad \forall s_m \in V_n . \tag{4}$$

This leads classically to the resolution of a system of linear equations.

2.2 Transport problem

In this paper, the concern is not the solving of the transport equation numerically. The method used here is the one described in [1]. One can refer to other methods such as discontinuous Galerkin or Lesaint-Raviart [3].

3 Conservation of mass

Classical finite element methods do not ensure the conservation of the resin mass when formulation (4) is used to solve the problem. The conservation equation is solved weakly i.e. the field $\mathbf{v} = K/\mu\,\mathrm{grad}\,p$ is not in its natural function space where the normal component of $\rho\mathbf{v}$ is continuous. In further examples, we will quantify this resin loss for 2-D and 3-D problems.

3.1 Possibility of div-conform approximation

A *div-conform* approximation is in :

$$H(\mathrm{div}, \Omega) = \left\{ \mathbf{v} \in \mathbf{L}^2(\Omega) ; \mathrm{div}\ \mathbf{v} \in L^2(\Omega) \right\}.$$

A sufficient condition for this is that $\mathbf{v}\cdot\mathbf{n}$ be continuous everywhere. We consider a mesh M of $\#E$ elements, $\#F$ faces and $\#N$ nodes. Using this mesh, we consider the set $S^0(M)$ of all the approximation spaces where $\mathbf{v}$ is constant in each element. With this assumption, it is clear that $\mathbf{v}\cdot\mathbf{n}$ is continuous everywhere except on the faces between elements. For the

approximation to be *div-conform*, $\#F$ constraints have to be imposed for ensuring the *div-conformity* (continuity of $\mathbf{v}\cdot\mathbf{n}$).

It is clear that the classical nodal approximation space $V_n \in S^0(M)$ cannot be *div-conform* Its dimension dim $V_n = \#N$ is too small to provide so much constrains: $\#N$ is always less than $\#F$ (in 2-D, $\#F \approx 3\#N$ and in 3-D, $\#F \approx 6\#N$). Note that the maximum size of an element of $S^0(M)$ is n $\#E$ where n is the dimension of the problem (or similarly the number of component for the velocity). In every case, n $\#E > \#F$. It is then affordable to build up such kind of *div-conform* approximation.

3.2 Discontinuous elements

The natural choice for *div-conform* element of $S^0(M)$ should be a space V_f of dimension dim $V_f = \#F$. In this case, we have exactly the good number of degrees of freedom in the interpolation to ensure the *div-conformity*.

Let us consider a mesh of tetrahedron. We construct the discrete approximation space as follow. For every face f of the mesh, we build up a linear scalar function s_f with support of the *2* adjacent elements of f. This function is equal to *1* on f and is equal to -2 on the *2* opposite nodes (Fig. 2).

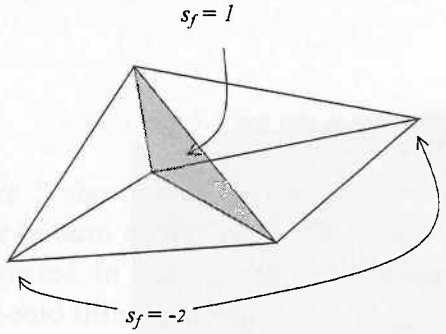

Fig. 2: Support of shape function s_f

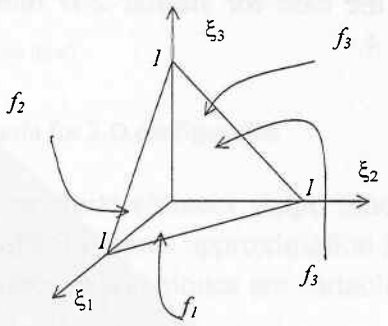

Fig. 3: Reference tetrahedron

If we consider the classical reference tetrahedron of Fig. 3, the local function space is of dimension *4*, one function s_f for every face f of the tetrahedron:

$$\begin{cases} s_1 = 1 - 3\,\xi_1 \\ s_2 = 1 - 3\,\xi_2 \\ s_3 = 1 - 3\,\xi_3 \\ s_4 = 3\left(\xi_1 + \xi_2 + \xi_3\right) - 2 \end{cases} \tag{5}$$

Choosing these shape functions for interpolating the pressure field do not ensure *a priori div-conformity.* Hence, we will shown that the finite element equations (4) are equivalent to the strong continuity of **u.n** when using the interpolation (5). Let us call e_1 and e_2 the support of function s_f (e_1 and e_2 share the same face f). Equation (4) for face f becomes:

$$\left(\mathbf{v},\, grad\; s_f\right)_{e_1} + \left(\mathbf{v},\, grad\; s_f\right)_{e_2} = 0\,.$$

If $\mathbf{v}_1$ is the constant on e_1 and $\mathbf{v}_2$ is constant on e_2, it is easy to demonstrate that:

$$\int_{e_1} grad\ s_f = \frac{2\,Volume(e_1)}{Height(e_1)}\mathbf{n}_f = a_f\ \mathbf{n}_f$$

where $\mathbf{n}_f$ is the normal to face f and a_f is the area of f. Equation (4) becomes simply :

$$(\mathbf{v}_1 - \mathbf{v}_2)\,\mathbf{n}_f\ a_f = 0$$

which imposes exactly the flux continuity i.e. *div-conformity*. Solution $\mathbf{v} = K^{-1}\text{grad}\,p$ of the finite element problem with discontinuous shape functions is proved to be *div-conform*. Note that the pressure field is discontinuous. If not, we would have obtained the exact solution of the given problem.

4 2-D versus 3-D

We present here a comparison of computations using classical continuous elements and new discontinuous elements for 2-D and 3-D configurations. The aim of this part is to show that 3-D continuous solutions are not acceptable for reasonable meshes. This is not the case for similar 2-D meshes. Let us consider the central injection problem of Fig. 4.

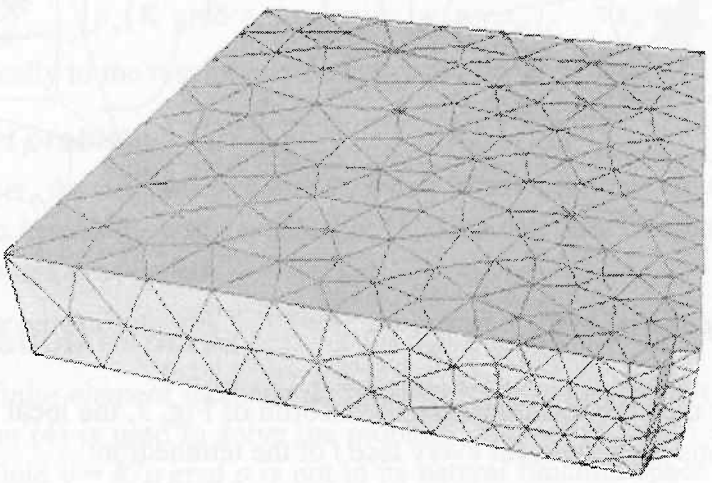

Figure 4. Mesh for the central injection.

This mesh is made of 2474 tetrahedron which is quite coarse. The problem is obviously two dimensional and we have also considered the upper part of the problem for a 2-D computation. The upper mesh is made of 244 triangles. Both 2-D and 3-D configurations were computed using classical nodal elements and discontinuous elements. Next figure shows front flow rates and fluid losses for the 2-D calculation.

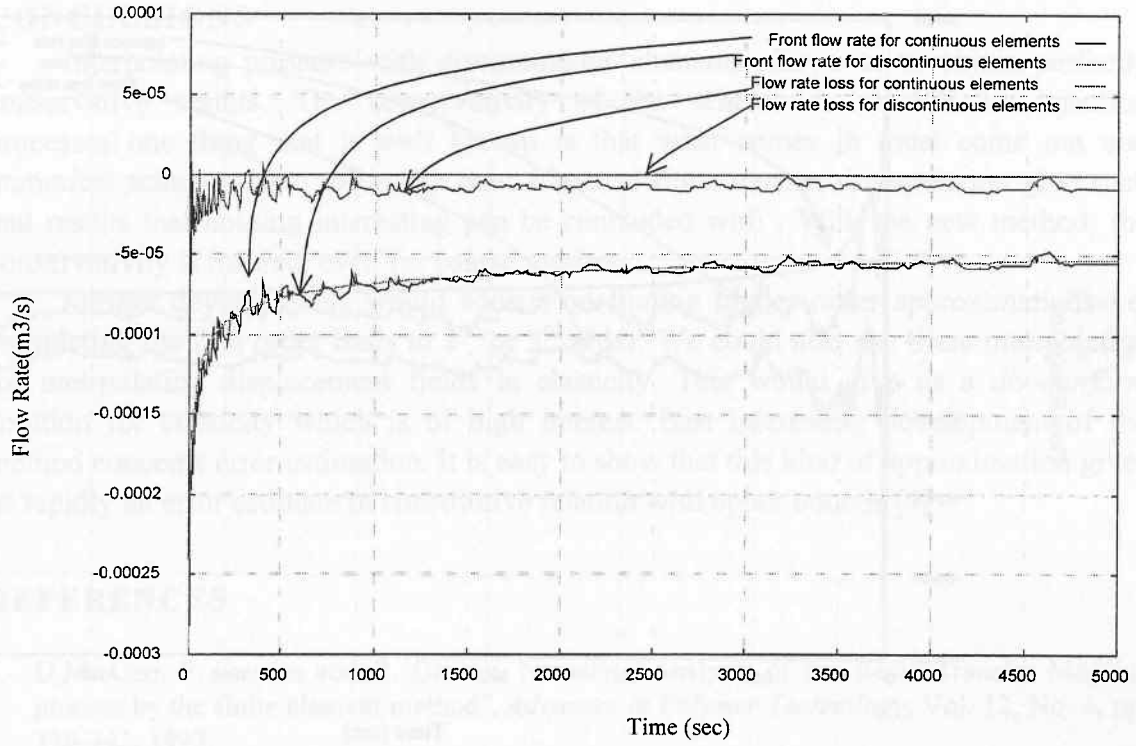

Fig 5: Flow rate at the front and losses of resin for 2-D configuration

Figure 5 shows clearly that the use of discontinuous finite element shape functions is better in term of flux conservativity but that classical continuous approximation is not to be rejected. In fact, it is known that classical finite element techniques are suitable in 2-D for mould filling problems[1].

Now, let us consider the 3-D problem. The 2-D and the 3-D mesh can be considered as equivalent in term of mesh density. Figure 6 presents, for classical continuous elements, flow rates at injection port and at front and the loss i.e. the difference between injection and front. It is clear that we cannot consider the solution as acceptable. The loss is of the same order of magnitude as the injection flow rate. The difference between 2-D and 3-D computations is there: for similar meshes, 2-D can give acceptable results while 3-D cannot. We could refine the mesh until the flow rate is sufficiently balanced (this balance is an excellent error estimate for the computation) but the number of time steps increase with the number of elements and so does the computation time for one time step. Using classical finite elements is definitely not a good solution. Figure 7 shows similar results for discontinuous elements. Perfect conservation of the flux is observed.

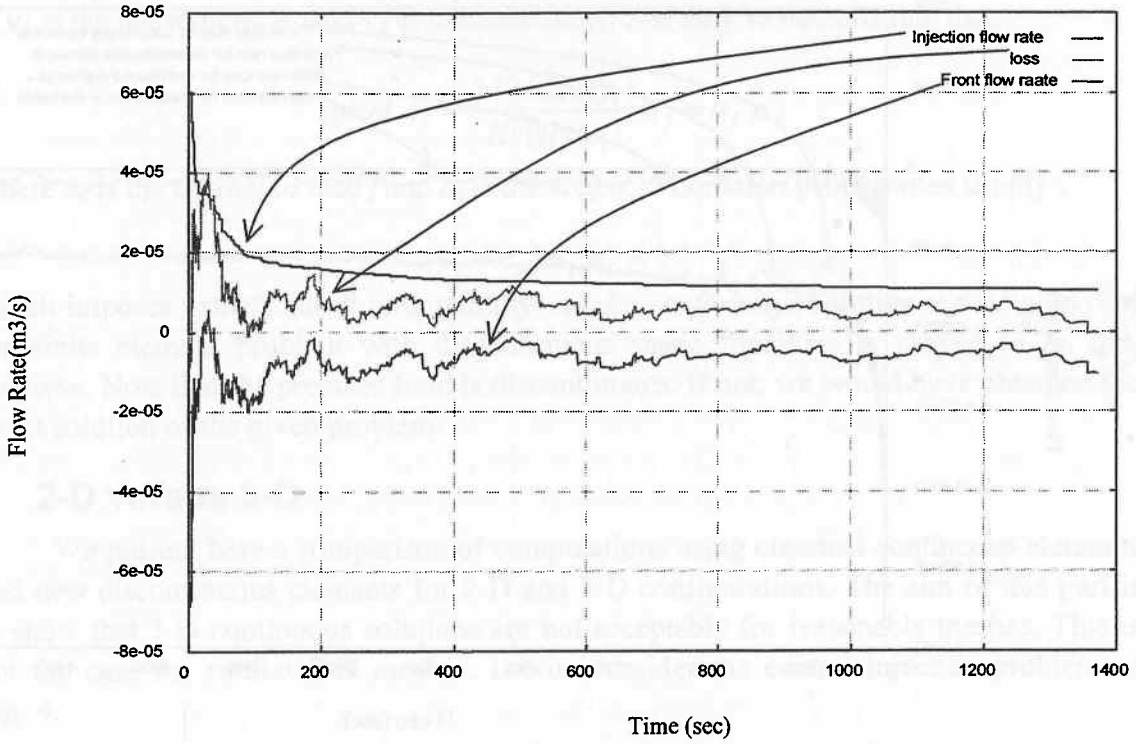

Fig 6: Flow rate at the front and at the injection and losses of resin for 3-D configuration with continuous elements

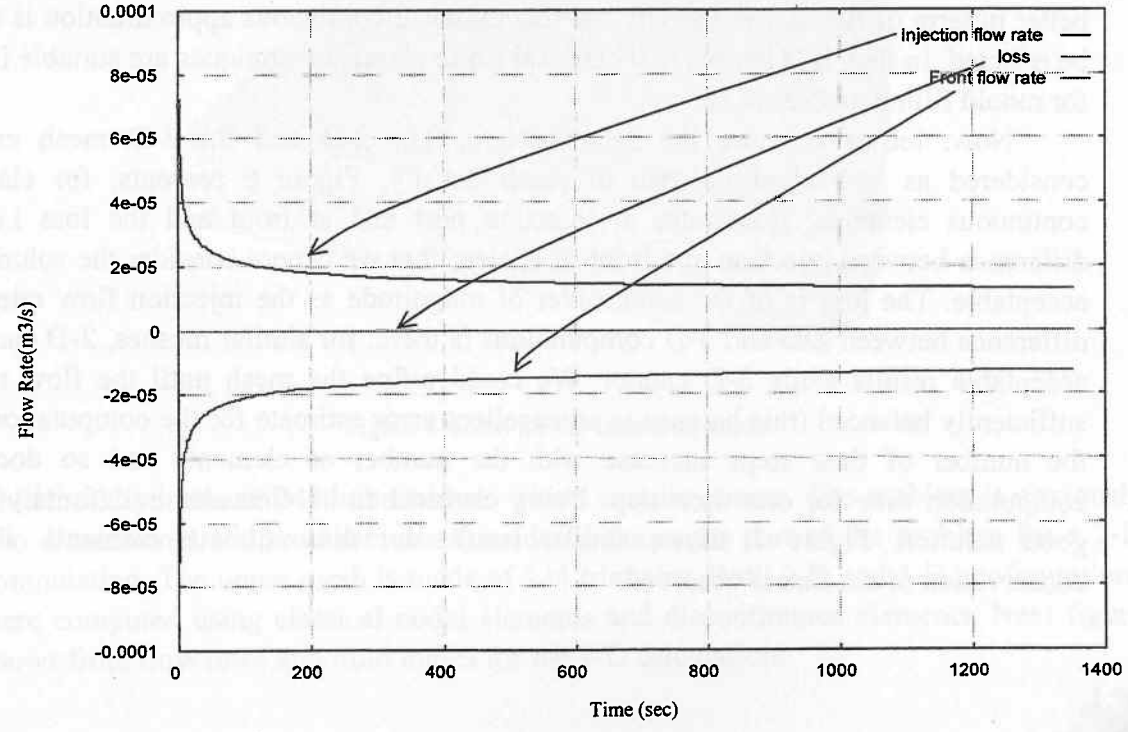

Fig 7: Flow rate at the front and at the injection and losses of resin for 3-D configuration with discontinuous elements

CONCLUSIONS

Interpolating pressure with discontinuous elements allows us to obtain perfectly conservative results. The conservativity of the scheme is crucial for injection processes: one thing that is well known is that what comes in must come out and numerical schemes have to be like that. Classical finite element interpolation give such bad results that nothing interesting can be concluded with . With the new method, the conservativity is insured, even for coarse meshes.

Further developments would consist on finding higher order approximations i.e. completing the first order basis to 2^{nd} or 3^{rd} order. We could also use these interpolation for inetrpolating displacement fields in elasticity. This would give us a *div-conform* solution for elasticity which is of high interest. Last interesting development of the method concerns error estimation. It is easy to show that this kind of approximation gives us rapidly an error estimate in constitutive relation with upper bounds [4].

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