Flow Processes in Composite Materials '94
University College Galway

RHEOLOGICAL MODEL FOR THE FORMING OF FABRIC REINFORCED THERMOPLASTIC SHEETS

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ABSTRACT

The paper investigates the suitability of the idealised fibre reinforced fluid (IFRF) model for the thermoforming of fabric reinforced thermoplastic sheets, and a strategy is proposed for determining the materials parameters required to characterise the sheet rheological behaviour. The IFRF theory for a viscous fluid with two inextensible directions is developed for modelling fabric sheets and specific forms of the constitutive equation are derived. Some simple flows are analysed and it is shown that in through-thickness shear flows, as for example in a torsion rheometer experiment, the fabric angle ϕ remains constant, whereas in in-plane flows ϕ is a function of the strain rate. Trellis deformations are investigated by considering the in-plane stretching flow of a fabric with fibres inclined to the load direction. The torsion rheometer test is analysed for a fabric pre-deformed to a fabric angle ϕ. In this case tests on rectangular specimens with different aspect ratios and fabric angles are proposed which enable the three viscosities in the model to be determined.

1. INTRODUCTION

There is considerable interest in fibre reinforced thermoplastic (FRTP) materials for aerospace and automotive components and a number of thermoplastic prepreg materials are commercially available with both glass and carbon fibre reinforcement. These materials have advantages over traditional thermosetting prepregs in that components may be thermoformed with short cycle times by hot stamping or diaphragm forming. The practical difficulties of obtaining successful thermoformed components has led to a requirement for understanding the rheological properties of FRTP sheets and for the development of computational techniques for process simulation based on these properties. One of the first high performance FRTP prepregs was APC-2 [1] in which unidirectional (UD) carbon fibres with volume fractions of about 60% reinforce a PEEK matrix. Processing of these materials takes place by diaphragm forming or rapid stamping at temperatures of 360 - 400°C at which the FRTP sheet consists of very stiff carbon fibres embedded in a soft viscous matrix. The rheological properties of APC-2 sheets are well characterised and successful process models have been developed based on constitutive equations for an idealised fibre reinforced fluid (IFRF). These rheological models are derived from a general theory for idealised fibre reinforced materials developed by Spencer [2] and his co-workers in which elastic, viscoelastic or plastic materials are reinforced by inextensible fibres. More recently Rogers [3] has developed a viscous form of the theory for modelling FRTP prepregs which has been successfully applied to a number of thermoforming problems for UD reinforced sheets. This fibre reinforced fluid model and its application in finite element (FE) programs for process simulation are reviewed in some detail in [4] and [5].

Since most structural components require fibre reinforcement in more than one direction and because it is labour intensive to stack up to 32 UD plies with different orientations for a 4 mm wall thickness, FRTP sheets with fibre fabric reinforcement have been developed. Recent examples are Cetex [6], which is a carbon fabric reinforced PEI resin for aerospace applications and Vestopreg [7], a glass fabric reinforced PA resin developed for automotive applications. Models for fabric reinforced materials are discussed in [8] - [12], where it is shown that the dominant in-plane deformation mechanism is the 'trellis effect' in intraply shear in which the fabric angle between the warp and weft directions changes. Secondary deformation mechanisms are fibre straightening and fibre extension under tensile forces. Simulation software based on an FE program which models the trellis effect has been developed in [8] and applied to model the drapability of fabrics. The trellis model is purely kinematic and consists of a mapping of the fabric geometry from an initial to a final surface, with intraply shearing as the only deformation mechanism. In [11] a similar program which also includes fibre extension has been used for the forming of fabric reinforced thermoplastic sheets, based on an FE model for a fabric as a network of elastic beam elements connected at nodal points. During intraply shear the fabric angle between the initially orthogonal fibres is reduced until an angle is reached at which the fabric locks or starts to buckle out-of-plane. Thus the main materials input data for these models is the locking angle of the fabric which limits the intraply shear deformation.

None of these fabric models includes the influence of the viscous matrix. Thus they cannot take account of the influence of processing conditions such as temperature, rate of loading, contact forces, interply friction and cooling effects which occur in the thermoforming process of FRTP materials. The aim of this paper is to propose a rheological model for fabric reinforced thermoplastic sheets which includes the matrix viscosity and which could be used in process simulation software. This is based on the extension of the successful IFRF for UD sheets in [3] to viscous sheets with two families of inextensible fibres. The fibre kinematics are already included in the IFRF constraint and constitutive equations, since the fibres are inextensible in the two main directions in the fabric, they are convected with fluid elements and can rotate in relation to each other in intraply shear deformations. Thus the model combines the important kinematic effects of the fabric model with a viscous fluid matrix.

The starting point of the present investigation in Section 2 is a general constitutive equation for an idealised material reinforced by two families of inextensible fibres given by Spencer [2]. A specific form of the theory suitable for modelling a 2-D fabric with a viscous matrix, which contains three independent viscosity parameters, is derived. Section 3 considers in-plane flows in the model and looks particularly at the influence of kinematics on fibre rotations. An exact solution is given for a fabric with fibres inclined at angles $\pm \frac{1}{2}\varphi$ to a tensile load, which models the trellis deformation effect. Measurement of $\phi(t)$ during the test could provide a method of determining the viscosity parameters. Torsion rheometer tests are currently favoured for measuring the two viscosities for UD FRTP sheets. In Section 4 this test is analysed for the fabric model. It is shown that torsional rheometer tests on 0/90 fabrics are not sufficient to determine the three viscosities and that tests on a deformed fabric with fabric angle $\phi < 90°$ are required. Section 5 concludes with a discussion of the validity of the fabric model presented here as a basis for the simulation of the thermoforming process for fabric reinforced FRTP sheets.

2. IDEALISED VISCOUS FLUIDS WITH TWO FAMILIES OF FIBRES

2.1 General formulation

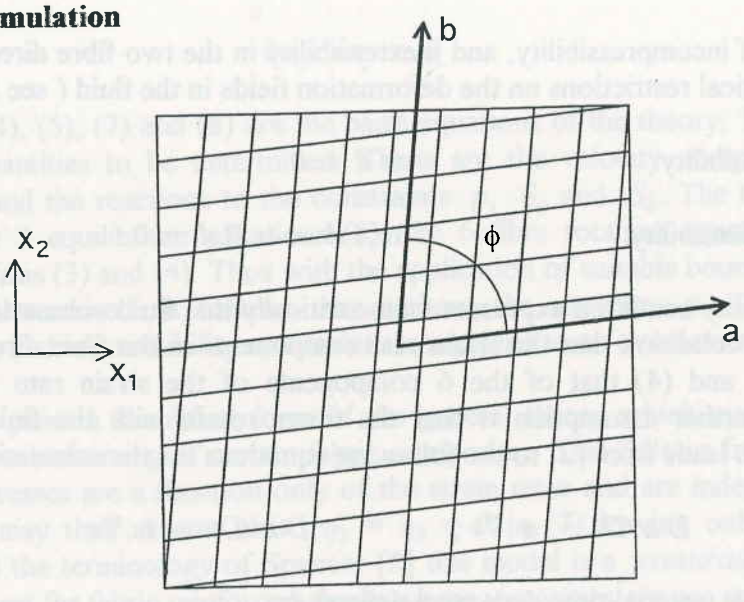

Fig. 1 Notation for the deformed fabric reinforced sheet

The fabric reinforced composite sheet is modelled as a continuum with continuously distributed fibres. Each ply consists of an incompressible, anisotropic Newtonian viscous fluid reinforced by two families of high stiffness fibres, which are assumed to be inextensible. The fibre directions in the fabric are denoted by unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, which are mechanically equivalent so that the fabric has identical properties in the two fibre directions. The fabric angle is denoted by ϕ so that $\cos\phi = \boldsymbol{a} \cdot \boldsymbol{b}$, the scalar product of the fibre directions. At the start of a thermoforming operation the fabric will usually have orthogonal fibres, but during forming the fibres are convected with the fluid causing them to rotate so that in general they will be inclined at some angle $\phi < 90°$, as shown schematically in Fig. 1. The derivation of the general inextensible constraint equations and the constitutive equations is described in [2] and [3], and we adopt similar notation and conventions here. Vector and tensor components are referred to a system of rectangular Cartesian coordinates x_i $(i = 1, 2, 3)$. The bold script is used for vector and tensor quantities and we adopt both an indicial and a whole vector notation for vectors and tensors, as appropriate. The motion of the continuum is described by the velocity vector $\boldsymbol{v}$ and the fibre orientation vectors $\boldsymbol{a}$, $\boldsymbol{b}$, which are in general functions of position $\boldsymbol{x}$ and time t. The appropriate kinematical quantity for describing the flow of a fabric reinforced sheet is the Eulerian rate of strain tensor $\boldsymbol{d}$ defined by

$$\boldsymbol{d} = \tfrac{1}{2}\left(\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^{\mathrm{T}}\right)$$

where ∇ is the derivative with respect to position $\boldsymbol{x}$ so that in index notation

$$d_{ij} = \tfrac{1}{2}\left(\partial v_i/\partial x_j + \partial v_j/\partial x_i\right) \tag{1}$$

Other convenient quantities are the tensors $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ defined in terms of the dyadic product of two vectors $\boldsymbol{a}\,\boldsymbol{b} = (a_i b_j)$ as

$$\boldsymbol{A} = \boldsymbol{a}\,\boldsymbol{a}, \qquad \boldsymbol{B} = \boldsymbol{b}\,\boldsymbol{b}, \qquad \boldsymbol{C} = \tfrac{1}{2}(\boldsymbol{a}\,\boldsymbol{b} + \boldsymbol{b}\,\boldsymbol{a})(\boldsymbol{a}\cdot\boldsymbol{b}) \tag{2}$$

We introduce the trace of a tensor $\operatorname{tr} \mathbf{T} = T_{ii} = T_{11} + T_{22} + T_{33}$, and note that since $\boldsymbol{a}$ and $\boldsymbol{b}$ are unit vectors $\operatorname{tr} A = \operatorname{tr} B = 1$.

The constraints of incompressibility, and inextensibility in the two fibre directions, leads to the following kinematical restrictions on the deformation fields in the fluid (see Spencer [2]:

$$\text{Incompressibility:} \qquad \operatorname{tr} \boldsymbol{d} = 0 \tag{3}$$

$$\text{Fibre inextensibility:} \qquad \operatorname{tr} \boldsymbol{A}\, \boldsymbol{d} = \operatorname{tr} \boldsymbol{B}\, \boldsymbol{d} = 0 \tag{4}$$

The incompressibility condition expresses mathematically that fluid volumes are preserved, and the inextensibility condition that the strain rate components in the fibre directions are zero. It follows from (3) and (4) that of the 6 components of the strain rate tensor only 3 are independent. A further assumption is that the fibres rotate with the fluid elements during deformation which leads from [2] to the following equations for the orientation vectors :

$$D\,\boldsymbol{a}/Dt = \boldsymbol{a}\,.\nabla \boldsymbol{v}\,, \qquad D\,\boldsymbol{b}/Dt = \boldsymbol{b}\,.\nabla \boldsymbol{v} \tag{5}$$

where D / Dt is the material time derivative defined as

$$D\,g/Dt = \partial g/\partial t + \boldsymbol{v}\,.\nabla g \tag{6}$$

The fabric model is completed by adding to the kinematical equations (1), (3), (4) and (5) constitutive equations which define the Cauchy stresses $\boldsymbol{\sigma}$ in the material in terms of the strain rates $\boldsymbol{d}$ and the fibre directions $\boldsymbol{a}$ and $\boldsymbol{b}$. For a fabric reinforced sheet under thermoforming conditions we require the general form of constitutive equation for a viscous fluid with two families of inextensible fibres. Such an equation for elastic materials is given by Rogers [3], from which the following general form for a Newtonian viscous fluid can be deduced on applying the viscoelastic correspondence principle :

$$\boldsymbol{\sigma} = -p\,\boldsymbol{I} + S_a\,\boldsymbol{A} + S_b\,\boldsymbol{B} + 2\eta_1\,\boldsymbol{d} + 2\eta_2\,(\boldsymbol{A}\,\boldsymbol{d} + \boldsymbol{d}\,\boldsymbol{A} + \boldsymbol{B}\,\boldsymbol{d} + \boldsymbol{d}\,\boldsymbol{B}) + 2\eta_3\,(\boldsymbol{C}\,\boldsymbol{d} + \boldsymbol{d}\,\boldsymbol{C}) \tag{7}$$

The kinematic constraints (3) and (4) have caused three arbitrary terms to appear in the constitutive equation which are not determinate from the deformation, these are the hydrostatic pressure p and the fibre tension stresses S_a and S_b, which are the reactions to the inextensibility constraints in the fibre directions.

This general constitutive equation contains three independent viscosity parameters η_1, η_2 and η_3, the first two of which are related to shear along and transverse to the fibres. We note from (2) that $C = 0$ when $\boldsymbol{a}\,.\,\boldsymbol{b} = 0$, ie when $\boldsymbol{a}$ is perpendicular to $\boldsymbol{b}$. It follows that the third viscosity η_3 contributes to the stresses only when the fibres are non-orthogonal and so is related to trellis deformations in the fibre plane where the fabric angle ϕ changes. Equation (7) is the most general invariant form for the stress tensor as a function of $\boldsymbol{d}$, $\boldsymbol{a}$, $\boldsymbol{b}$, which is linear in the strain rate tensor $\boldsymbol{d}$. We note from [2] that the quantity $(\boldsymbol{a}.\boldsymbol{b})^2 = \cos^2\phi$ is also a possible scalar invariant so that the viscosities η_1, η_2 and η_3 could in general be functions of $\cos^2\phi$. For 0/90 fabrics $\cos\phi = 0$ and $\cos^2\phi$ remains small for fabrics with fibre angles above 65°. In this paper we consider a first order theory and assume that the viscosities are materials constants independent of fabric angle ϕ.

The thermoforming model for fabrics is now completed by the equations of motion. Sheet forming is a relatively slow forming process and if we assume that dynamic inertia effects are negligible in the deforming sheet, then in the absence of body forces the stress $\boldsymbol{\sigma}$ must satisfy the equilibrium equations which in component form are :

$$\partial \sigma_{ij} / \partial x_j = 0 \qquad (8)$$

Equations (3), (4), (5), (7) and (8) are the basic equations of the theory. There are essentially 12 unknown quantities to be determined. These are the velocity components **v**, the fibre directions **a**, **b** and the reactions to the constraints p, S_a and S_b. The theory has 12 scalar equations in the 3 equilibrium equations (8), the 6 fibre rotation equations (5) and the 3 constraint equations (3) and (4). Thus with the application of suitable boundary conditions, the theory is well determined. It follows that without the reactions p, S_a and S_b , the incompressibility and inextensibility constraints would lead to an overdetermined theory.

We note at this point a simplified form of the general theory which may be of interest for modelling the thermoforming of certain fabric composites. This follows from (7) on assuming that the extra stresses are a function only of the strain rates and are independent of the fibre directions. We may thus assume that $\eta_2 = \eta_3 = 0$ in (7) leaving only a single viscosity parameter η_1. In the terminology of Spencer [2] this model is a *constrained viscous fluid*. It could be of interest for fabric reinforced sheets whose viscosities are not strongly influenced by the presence of the fibres, ie the longitudinal. transverse and trellis shear viscosities are the same, which may be the case for loosely woven fabrics or FRTP sheets with high resin contents.

2.1 IFRF model for fabric reinforcement

In order to see whether (7) is a suitable constitutive equation for modelling FRTP fabric composites, simple flow solutions need to be investigated and compared with experimental observations, and test methods are required for determining the three viscosity parameters. Specific forms of the constitutive equation can be written down depending on the choice of coordinate system and the fibre direction vectors $\boldsymbol{a}$, $\boldsymbol{b}$. In general fabrics of interest have orthogonal families of fibres. However, during thermoforming the fibres may rotate and the fibre angle could change from the initial value of 90°. It follows that the required form of the model should be for non-orthogonal fibres with general fabric angle φ and contain the three viscosity parameters, since the reduced theory for orthogonal fabrics with two viscosities is too restrictive. Note that several specific forms of the general theory may be required depending on the loading conditions of interest.. The one chosen here is convenient for analysing the trellis effect in fabric composites and for understanding certain torsional rheometer tests. Other forms of the general theory are discussed in [13] and applied to shear loading of fabrics.

Our interest is in thin fabric reinforced thermoplastic sheets in which the fibres lie in a plane, which is taken as the x_1-x_2 plane. Small out of plane fluctuations in the fibre directions due to the weave pattern are neglected. In this paper we choose coordinate axes which bisect the fibre directions in the fabric, thus the two families of fibres make angles $\pm\,\theta$ to the x_1-axis as shown schematically in Fig. 2 and the fabric angle $\varphi = 2\,\theta$. The fibre direction vectors are given by :

$$\boldsymbol{a} = (\cos\theta, \sin\theta, 0) \qquad \boldsymbol{b} = (\cos\theta, -\sin\theta, 0) \qquad (9)$$

where for families of straight fibres $\theta(t)$ is independent of position $\boldsymbol{x}$ in the sheet. We introduce the notation

$$m = \cos\theta(t), \qquad n = \sin\theta(t)$$

and may assume without loss of generality here that $m \neq 0$, $n \neq 0$ for all times t.

On substituting (9) into (7) we obtain the following explicit expressions for the stress components :

$$\sigma_{11} = -p + m^2 (S_a + S_b) + d_{11} [\, 2\eta_1 + 8 m^2 \eta_2 + 4 m^2 (m^2 - n^2)\, \eta_3 \,]$$

$$\sigma_{22} = -p + n^2 (S_a + S_b) + d_{22} [\, 2\eta_1 + 8 n^2 \eta_2 - 4 n^2 (m^2 - n^2)\, \eta_3 \,]$$

$$\sigma_{33} = -p + 2\eta_1 d_{33} \tag{10}$$

$$\sigma_{23} = d_{23} [\, 2\eta_1 + 4 n^2 \eta_2 - 2 n^2 (m^2 - n^2)\, \eta_3 \,]$$

$$\sigma_{13} = d_{13} [\, 2\eta_1 + 4 m^2 \eta_2 + 2 m^2 (m^2 - n^2)\, \eta_3 \,]$$

$$\sigma_{12} = mn (S_a - S_b) + d_{12} [\, 2\eta_1 + 4\eta_2 + 2 (m^2 - n^2)^2 \eta_3 \,]$$

Since our main interest is in the in-plane and through-thickness behaviour of thin FRTP sheets, it is convenient to write the constitutive equations in an alternative form. From the incompressibility condition (3)

$$d_{33} = -(d_{11} + d_{22}) \tag{11}$$

hence from $(10)_3$ the pressure is given by

$$p = -\sigma_{33} - 2\eta_1 (d_{11} + d_{22}) \tag{12}$$

and on substituting for the pressure in (10) the equations take the form :

$$\begin{pmatrix} \sigma_{11} - \sigma_{33} \\ \sigma_{22} - \sigma_{33} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{pmatrix} \begin{pmatrix} d_{11} \\ d_{22} \\ 2d_{12} \end{pmatrix} + \begin{pmatrix} m^2 (S_a + S_b) \\ n^2 (S_a + S_b) \\ mn (S_a - S_b) \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} \sigma_{23} \\ \sigma_{13} \end{pmatrix} = \begin{pmatrix} D_{44} & 0 \\ 0 & D_{55} \end{pmatrix} \begin{pmatrix} 2d_{23} \\ 2d_{13} \end{pmatrix} \tag{14}$$

where the viscosity coefficients D_{ij} are given by :

$$D_{11} = 4 [\, \eta_1 + 2 m^2 \eta_2 + m^2 (m^2 - n^2)\, \eta_3 \,]$$

$$D_{22} = 4 [\, \eta_1 + 2 n^2 \eta_2 - n^2 (m^2 - n^2)\, \eta_3 \,]$$

$$D_{12} = 2\eta_1 \tag{15}$$

$$D_{66} = \eta_1 + 2\eta_2 + (m^2 - n^2)^2 \eta_3 \,]$$

$$D_{44} = \eta_1 + 2 n^2 \eta_2 - n^2 (m^2 - n^2)\, \eta_3$$

$$D_{55} = \eta_1 + 2 m^2 \eta_2 + m^2 (m^2 - n^2)\, \eta_3$$

For thin sheets under plane stress conditions in the x_1-x_2 plane it is usual to assume that the through thickness stresses are zero in which case

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$$

and with $\sigma_{33} = 0$ equation (10) is the plane stress constitutive equation. The analysis of torsion rheometer tests is based on (11) and we cannot then assume plane stress conditions. The general form of equations (10) and (11) are thus preferred since they allow direct and shear contact stresses between FRTP sheets.

In addition to these constitutive equations the strain rate tensor and fibre orientation vectors must satisfy the inextensibility conditions (4) which take the specific forms :

$$m^2 d_{11} + 2\,mn\,d_{12} + n^2 d_{22} = 0, \qquad m^2 d_{11} - 2\,mn\,d_{12} + n^2 d_{22} = 0 \qquad (16)$$

and the two independent rotation equations :

$$Dm/Dt = m\,d_{11} + n\,\partial v_1/\partial x_2, \qquad Dm/Dt = m\,d_{11} - n\,\partial v_1/\partial x_2 \qquad (17)$$

Because $m^2 + n^2 = 1$ and the conditions (16), it can be shown that the two further equations in Dn/Dt are redundant. It will be shown in the following sections that the kinematical constraints (16) and (17) impose severe restrictions on possible flow fields in the model for fabric reinforced sheets.

It should be noted here that equations (10) - (17) are only valid for fabrics as shown in Fig. 2 and for flow fields in which the fibre directions remain at angles $\pm\,\theta$ (t) to the x_1-axis. For general flow fields this will not be the case and it would then be necessary to transform the spatial coordinates (x_1, x_2, x_3) to local coordinates which bisect the fibre directions in order to use (10) and (11) for the calculation of stresses. Other choices of the fibre directions are more suitable for particular flows and for implementation into FE programs, for example in the latter case it is convenient to use an element coordinate system with the x_1-axis as one fibre direction and the second direction in the x_1-x_2 plane at angle φ to the x_1-axis. This form of the general theory is developed further in [13].

3. IN-PLANE FLOWS

3.1 Stretching of a fabric sheet

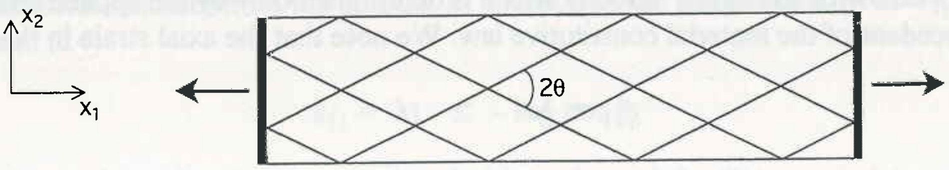

Fig. 2 Fabric sheet with fibres at $\pm\,\theta$ to the x_1-axis

In this section we analyse in-plane flows in the model proposed above for a fabric with fabric angle 2θ with the x_1-axis chosen as the bisector of the fibre directions, as shown schematically in Fig. 2. An exact solution to the governing equations is given which could be used to validate the model against the observed response of stretched fabric sheets subjected to trellis deformations, and as a possible basis for measuring the viscosity parameters.

We consider first the inextensibility constraint equations (16). These can be re-written as

$$m^2 d_{11} + n^2 d_{22} = 0, \qquad mn\, d_{12} = 0 \tag{18}$$

Since $m \neq 0$, $n \neq 0$ it follows that the in-plane shear rate $d_{12} = 0$. Because of the incompressibility condition (11) and $(18)_1$ it follows that only one of the three direct strain rates d_{11}, d_{22} and d_{33} is independent. Thus the most general in-plane flow of the fabric which satisfies the kinematic constraints has the form :

$$d_{22} = - d_{11}\, m^2 / n^2, \quad d_{33} = d_{11}\,(m^2 - n^2) / n^2, \quad d_{12} = d_{13} = d_{23} = 0 \tag{19}$$

and is thus characterised by the single strain rate component d_{11}. Note that this is the most general form of in-plane flow which satisfies the inextensibility and incompressibility constraints, and which preserves the balance of the fabric, ie that the fibres remain at $\pm\,\theta(t)$ to the x_1-axis for all times t, which is a severe restriction.

Our interest is in modelling the stretching of a fabric sheet as shown in Fig. 2. We consider the case of the sheet pulled at a constant velocity in the x_1 direction, thus the axial strain rate has a constant value $\lambda > 0$ and we may set

$$d_{11} = \lambda \tag{20}$$

On integration of (19) and (20) and assuming that the sheet in Fig. 2 is fixed at the left end which is also the origin of the coordinates, we obtain for the velocity field in the sheet

$$v_1 = \lambda x_1, \qquad v_2 = -\lambda x_2 \cot^2\theta, \qquad v_3 = \lambda x_3 \cos 2\theta / \sin^2\theta \tag{21}$$

Substitution of the velocity fields in (17) leads to a single differential equation for $\theta(t)$

$$d\theta / dt = - \lambda / \tan\theta \tag{22}$$

which has the solution

$$\cos\theta = \cos\theta_0 \exp(\lambda t) \tag{23}$$

where $\pm\,\theta_0$ are the initial fibre angles in the sheet at time $t = 0$. We note that (23) is even in θ so that it has roots in pairs $\pm\,\theta$ and it follows that during the stretching flow the fabric remains symmetric about the x_1-axis. It follows from (22) and (23) that when $\lambda > 0$ the fibres rotate towards the x_1-axis with an angular velocity which is determined only by the applied strain rate λ and is independent of the material constitutive law. We note that the axial strain in the sheet

$$\varepsilon_{11} = \lambda t \tag{24}$$

thus the sheet undergoes a trellis deformation with the fibre angle being dependent only on the axial extension of the sheet.

Next we consider what applied stresses are required to maintain this deformation field in the FRTP sheet. Substitution of (19) into (13) gives for the non-zero stresses :

$$\sigma_{11} - \sigma_{33} = \lambda D_{11} - \lambda D_{12}\, m^2/n^2 + m^2 (S_a + S_b)$$

$$\sigma_{22} - \sigma_{33} = \lambda D_{12} - \lambda D_{22}\, m^2/n^2 + n^2 (S_a + S_b) \tag{25}$$

$$\sigma_{12} = m\, n\, (S_a - S_b)$$

In the tensile test depicted in Fig. 2 loads at the end of the specimen cause uniaxial stresses σ_{11} in the sheet. On assuming there are no applied transverse and shear stresses we may set $\sigma_{12} = \sigma_{22} = 0$. We may also use $(10)_3$ and $(19)_2$ to define the pressure p so that the through-thickness stresses $\sigma_{33} = 0$. Hence $(25)_2$ and $(25)_3$ may be solved explicitly for the fibre tensions, giving

$$S_a = S_b = \lambda\, (m^2 D_{22} - n^2 D_{12}) / 2\, n^4 \tag{26}$$

and the only non-zero stress component is the applied axial stress, which may be expressed in terms of θ as :

$$\sigma_{11} = \lambda [\, 4\eta_1 - (3\eta_1 - 2\eta_2) \sin^2 2\theta - \tfrac{1}{4}\eta_3 \sin^2 4\theta\,] / \sin^4\theta \tag{27}$$

For the steady flow being considered the stresses are functions of time t but not of position x, thus the equilibrium equations (8) are satisfied automatically. It follows that the velocity field (21) and fibre orientation function θ(t) in (24), with the stresses (26) and (27), provide an exact solution in the model for the steady extension of a fabric reinforced sheet.

3.2 Validity of the fabric model

Next we consider whether the proposed theory is a suitable model for fabric reinforced thermoplastic sheets during thermoforming. This can be assessed by comparison of the predicted flow field with the observed or measured behaviour of fabric sheets. We consider a 0/90 fabric reinforced thermoplastic sheet loaded in tension along the bisector of the fabric angle in a displacement controlled test with a constant strain rate λ, as depicted in Fig. 2. Thus $\theta_o = 45°$ and from (22) in a tension test where $\lambda > 0$ we see that $d\theta / dt < 0$ and the fibre rotates towards the x_1-axis with an angular velocity which increases as θ is reduced, so that the fabric angle 2θ decreases with loading time. From (21) the stretching flow in the x_1 direction is accompanied by lateral contraction in the transverse x_2 direction, and for $\theta < 45°$ by an increase in sheet thickness. If $\lambda < 0$ as in a compression test the fabric angle increases, accompanied by an expansion in the transverse direction and a thickness reduction. Thus the flow field simulates the trellis deformation observed in fabrics. We note that there is a mathematical restriction on the amount of rotation in the fibres since $|\cos\theta| < 1$, thus from (23) and (24)

$$\varepsilon_{11} = \lambda t \;\; \leq \; -\log\cos\theta_o \tag{28}$$

and the tensile strain has a maximum value. However before this strain is reached in practice the fabric will certainly reach a locking angle ϕ^*.

The analysis shows that the tensile test proposed here is an excellent method of studying the trellis effect in fabric sheets. In practice it should be possible to perform the tests by placing the fabric prepreg specimen on a heated plate at the processing temperature and by applying the tensile load through grips attached outside the heated region. The fibre angle during deformation could then be observed from above. By measuring the rotation θ(t) during loading equation (23) could be validated as a first check on the model. The test can be continued up to the locking angle ϕ^* at which no further in-plane rotation is possible, which will be accompanied by out-of-plane buckling of the sheet, and/or a rapid increase in the applied load.

This minimum fabric angle is an important parameter for fabrics and test data obtained can then be used as a minimum allowable angle or failure condition during flow simulations based on the constitutive model proposed here. Data on the locking angle for dry fabrics obtained from shear tests in a picture frame rig are reported in [10] and [12]. This angle depends on several fabric parameters, such as fibre and weave type, reinforcement weight / unit area, etc. Typical reported values for glass fabric [10] are 72° (plain weave, 200 g/m²), 47° (satin, 300 g/m²), with lower values for carbon fabrics [12] of 28° (plain, 200 g/m²) and 25° (satin, 360 g/ m²). The data themselves are inconclusive and it is also unclear whether the presence of the thermoplastic matrix will act as a lubricant and lower the fabric angle, or whether it will restrict fibre rotation and hence increase the angle. Test data on fabric reinforced thermoplastic prepregs are thus required. They are also a means of bringing information on different fabric types into the model.

Finally, by measuring the longitudinal viscosity $\eta = \sigma_{11} / \lambda$ in the test some information can be obtained on the three viscosity parameters η_1, η_2 and η_3. Whether this test leads to a practical method of determining the viscosities is not yet clear. Even if these viscosities are material constants, as under constant temperature and rate conditions, we see from (27) that the longitudinal viscosity η is time dependent as a result of the fabric angle 2θ changing with time. In theory by measurement of $\theta(t)$ and the tensile load in a constant strain rate test, equation (27) may be used at three different times to give three linear equations for the determination of η_1, η_2 and η_3. Additional test data at other times can then be used to check the model. We note that for the constrained viscous fluid model $\eta_2 = \eta_3 = 0$ and the longitudinal viscosity is considerably simplified, but is still time dependent.

4. TORSIONAL FLOW IN THIN PLATES

Torsional viscometer test are well established [14] for measuring the longitudinal and transverse viscosities η_L and η_T of UD FRTP sheets at the forming temperature, using the test procedure and analysis method proposed in [3]. In this section we consider through-thickness shear flows in the fabric model and extend the torsional rheometer test analysis method to fabrics as a means of measuring the three viscosity parameters required in the model. The test set-up is shown schematically in Fig. 3. A thin rectangular plate specimen with sides length a, b and thickness h, having fibres inclined at angles $\pm\,\theta$ to the x_1-axis, is subjected to a steady or cyclic torsional flow by application of a torque on the upper plate of the rheometer about the x_3-axis. A transducer measures the torque applied on the fixed lower plate. For thin specimens with possible edge effects neglected the flow field is essentially pure torsion. As discussed in [3], if the lower plate is fixed and the upper plate rotates with constant angular velocity Ω, the velocity field is :

$$v_1 = -\,\Omega\, x_2\, x_3 / h, \qquad v_2 = \Omega\, x_1\, x_3 / h, \qquad v_3 = 0 \tag{29}$$

It follows from (1) that the strain rate components are :

$$d_{13} = -\,\tfrac{1}{2}\,\Omega\, x_2 / h, \quad d_{23} = \tfrac{1}{2}\,\Omega\, x_1 / h, \quad d_{11} = d_{22} = d_{12} = d_{33} = 0 \tag{30}$$

and the incompressibility condition (3) and inextensibility conditions (16) are satisfied identically. The flow field is a through-thickness shear of the FRTP sheet with no in-plane extension or shear component.

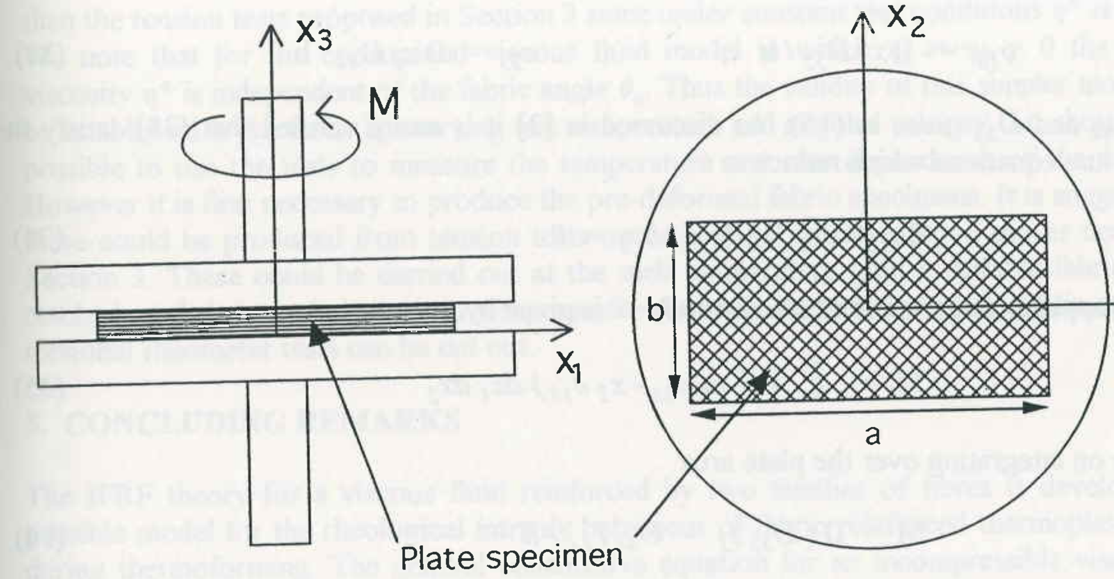

Fig. 3 Schematic diagram of the torsional rheometer test

The fibre rotation equation $(17)_1$ for the $+\,\theta$ fibres now becomes

$$d(\cos\theta)\,/\,dt \;=\; -\,\Omega\, x_3 \sin\theta\,/\,\mathrm{h} \qquad (31)$$

and since we assume that $\theta \neq 0$ this reduces to :

$$d\theta\,/\,dt \;=\; \Omega\, x_3\,/\,\mathrm{h}$$

Equation (31) is an odd function of θ and on setting $\theta \rightarrow -\,\theta$ it becomes identical to $(17)_2$, which is therefore automatically satisfied. The solutions of (31) for the two fibre directions are therefore :

$$\theta \;=\; \pm\,\theta_o + \Omega\, x_3\, t\,/\,\mathrm{h} \qquad (32)$$

where θ_o is the fabric angle at $t = 0$. It follows that the two families of fibres rotate together with the same angular velocity given in (31), and that the fabric angle $\phi = 2\,\theta_o$ remains constant during the test. Each plane of the plate through the thickness rotates with a different angular velocity, varying linearly from 0 to Ω between the baseplate and the top plate. Note that the angular velocity is kinematically determined and independent of the constitutive equations for the material.

Thus in a torsional flow the fabric angle does not change and the trellis deformation is no longer observed. A further consequence of this rotation of the fabric is that for times $t > 0$ the x_1 coordinate axis no longer bisects the fibre directions in the fabric except on the plane $x_3 = 0$. It follows that the constitutive equations (13) and (14) are only valid for calculating the stresses on the base plate where the fibre angles remain at $\pm\,\theta_o$. In order to use these constitutive equations for calculating the stresses on planes $x_3 > 0$ it is necessary to use transformed coordinates which rotate with the material on each plane through the thickness.

Next we consider the applied torque on the base plate. On assuming there are no applied in-plane stresses in the plate and since the in-plane strain rate components in (30) are zero, it follows from (13) that $S_a = S_b = \Gamma = 0$ and the only non-zero stress components are :

$$\sigma_{13} = -\,\Omega\, x_2 D_{55} / h \;, \qquad \sigma_{23} = \Omega\, x_1 D_{44} / h \tag{33}$$

with D_{44} and D_{55} given in (15). As discussed in [3] it is easily verified that (33) satisfy the equilibrium equations which reduce to :

$$\partial\sigma_{13} / \partial x_1 + \partial\sigma_{23} / \partial x_2 = 0 \tag{34}$$

and the applied torque on the base plate $x3 = 0$ is given by :

$$M = \int\!\!\int (x_1 \sigma_{23} - x_2 \sigma_{13})\, dx_1\, dx_2 \tag{35}$$

whence on integrating over the plate area

$$M = \Omega\,(D_{55} I_1 + D_{44} I_2) / h \tag{36}$$

where I_1 and I_2 are the second moments of area of the plate section. For a rectangular plate with sides a, b this may be written :

$$M = a\, b^3\, \Omega\, \eta^* / 12\, h \tag{37}$$

where η^* is the torsional viscosity

$$\eta^* = D_{55} + (a^2 / b^2)\, D_{44} \tag{38}$$

On substituting for D_{44} and D_{55} from (15), noting that on the base plate θ remains constant at $\theta = \theta_o$, and on setting the square of the plate aspect ratio c = a^2 / b^2, we obtain for the torsional viscosity :

$$\eta^* = \eta_1 (1+c) + 2\,\eta_2 (\cos^2\theta_o + c \sin^2\theta_o) + \eta_3 \cos 2\theta_o (\cos^2\theta_o - c \sin^2\theta_o) \tag{39}$$

We see from (39) that measurement of the torsional viscosity η^* on three different plate specimens enables in principle the three viscosities η_1, η_2 and η_3 to be determined. For an orthogonal fabric $\theta_o = 45°$ hence $\cos 2\theta_o = 0$ and the η_3 contribution to η^* is zero. Since the fabric angle remains constant in the test, it is thus not possible to measure all the viscosities on a 0/90 fabric. Test on a pre-deformed fabric sheet with fabric angle $\varphi = 2\theta_o < 90°$ are thus necessary. A possible test programme might consist of the following torsional rheometer tests :

Standard prepreg - fabric angle $\varphi = 90°$:

Square plate a/b = 1 :

$$\eta^* = 2\,(\eta_1 + \eta_2) \tag{40}$$

Deformed prepreg - fabric angle $\varphi = 2\theta_o < 90°$:

Square plate a/b = 1 :

$$\eta^* = 2\,(\eta_1 + \eta_2) + \eta_3 \cos^2 2\theta_o \tag{41}$$

Rectangular plate a/b = 2 :

$$\eta^* = 5\eta_1 + 2\,\eta_2 (\cos^2\theta_o + 4 \sin^2\theta_o) + \eta_3 \cos 2\theta_o (\cos^2\theta_o - 4 \sin^2\theta_o) \tag{42}$$

The results of the first two tests on square plates enables $(\eta_1 + \eta_2)$ and η_3 to be determined. The third test on a rectangular plate then gives a further linear relation between η_1 and η_2. Since θ_o remains constant during the tests, the test procedure should be easier to carry out than the tension tests proposed in Section 3 since under constant test conditions η^* is constant. We note that for the constrained viscous fluid model in which $\eta_2 = \eta_3 = 0$ the torsional viscosity η^* is independent of the fabric angle θ_o. Thus the validity of this simpler model could be established in the tests. By varying test temperature and angular velocity Ω it should also be possible to use the tests to measure the temperature and rate dependence of the viscosities. However it is first necessary to produce the pre-deformed fabric specimens. It is suggested that these could be produced from tension tests on orthogonal fabrics carried out as described in Section 3. These could be carried out at the melt temperature until a suitable fabric angle is reached, and then cooled to room temperature where the rectangular plate specimens for the torsional rheometer tests can be cut out.

5. CONCLUDING REMARKS

The IFRF theory for a viscous fluid reinforced by two families of fibres is developed as a possible model for the rheological intraply behaviour of fabric reinforced thermoplastic sheets during thermoforming. The general constitutive equation for an incompressible viscous fluid matrix reinforced by two families of inextensible fibres contains three viscosity parameters and two indeterminate fibre tension functions, in contrast to the two viscosities and single tension function of the established rheological model for UD FRTP sheets [4], [5]. The proposed fabric model is idealised since it takes no account of the fabric weave and does not include any interaction effects between the warp and weft fibres. It models the influence of the forming flows on the fibre directions and the kinematic effects of the assumed inextensible fibres on possible deformation and flow fields. It extends earlier kinematic models for fabrics [8]-[10] to include a viscous matrix. Interaction between fibres is included in such continuum models through their influence on the viscosity parameters and on the locking angle of the fabric φ^*, which require measurement for different fabric reinforced prepregs. The value of φ^* could then be a minimum allowed fabric angle for the validity of the constitutive law, and the attainment of this angle in an element during forming could be a possible failure criterion in the model. Viscoelastic and thermal effects which may be important on thermoforming of fabric prepreg could be included in the theory to first order by allowing the viscosity functions to be rate and temperature dependent.

The constitutive equations now require validation by careful comparison of predicted flow fields and fibre orientations with the observed behaviour of thermoplastic prepreg with fabric reinforcement. A specific form of the theory is given for fabric sheets reinforced in the x_1-x_2 plane with fibres inclined at angles $\pm\,\theta$ to the x_1-axis. This is convenient for the analysis of tension tests and torsional rheometer tests on fabric reinforced sheets. Both types of test would give valuable information on the validity of the model and could be used as a basis for measuring the three viscosities in the theory. It is shown that during in-plane loading of fabric reinforced sheets the fabric angle will change with time under load, which complicates the analysis of the test for determining sheet viscosities. By contrast in a torsion rheometer test the fabric angle is predicted to remain constant during the test. However, a consequence is that tests on a pre-deformed fabric are required to obtain all three viscosities. Other specific forms of the theory are more suitable for modelling in-plane shear tests on fabrics, or for incorporation into FE forming software, and these are discussed further in [13].

Acknowledgements : Part of the work was carried out within an EU funded BRITE project BE-509 and the author wishes to acknowledge the support received from the following project partners: Dassault Aviation, Dornier Luftfahrt, Engineering Systems International, EPFL Lausanne, Sulzer Innotech, University College Galway and University of Limerick.

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